Pension Reform, Assets Returns and Wealth Distribution

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Abstract

Does a pension reform exacerbate inequality in modern economies? Can a pension reform be positive for all agents? Conventional wisdom is that in the OLG framework a PAYG pension system generates lower aggregate savings, and thus a lower capital accumulation and therefore a higher interest rate. But the PAYG system is viewed as being more inequality-reducing than a privately funded system.

Our paper points out the importance of the heterogeneity of assets earnings and the unequal access that individuals have to them dependent on their wealth inheritance. In this paper there are two assets; one yielding the equilibrium interest rate as return and the other a high return. There is also a wealth threshold separating the agents into poor-constrained lender that earn the interest rate and rich-unconstrained borrower that earn the high return. In this context, the PAYG pension system generates more constrained-poor agents and a lower interest rate. That is the PAYG system generates more inequality and decreases the interest rate. The rich agents benefits from the reform at the expense of poor agents.

Keywords: Pension reform, Inequality, Wealth distribution, Assets returns

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1 INTRODUCTION

Does a pension reform exacerbate inequality in modern economies? Can a pension reform be positive for all agents? What is the effect of the pension system and its’ size on the credit market interest rate and on the economic dynamic? These questions are about the redistributive and efficiency aspects of any pension system reform, and they have been largely documented in the literature.

Conventional wisdom is that in the OLG framework a PAYG pension system generates lower aggregate savings, and thus a lower capital accumulation and therefore a higher interest rate. But the PAYG system is viewed as being more inequality-reducing than a privately funded system. Indeed, PAYG systems redistribute income across generations, but also within generations as the benefits accruing to an individual are not proportional to the taxes he pays. Deaton, Gourinchas and Paxson (2000) show that as PAYG system shares individual risk in earnings or asset returns across individuals, it moderates inequality in the economy. However these views have been challenged in the literature3, Cubeddu (2000) shows that in the presence of perfect insurance markets unfunded pension schemes are welfare reducing by essentially forcing individuals to substitute private assets for social security tax contributions. Since in dynamically efficient economies, the return on unfunded pension schemes is less than the return on private saving. The magnitude of the loss depends on life expectancy and labor ability. Moreover, Hairault and Langot (2002) showed that in an economy with a high heterogeneity in wealth a PAYG pension system may be inequality enhancing. The poor workers are then the losers in the PAYG system.

These analyses seem to miss an important aspect of the relationship between pension system and inequality. Namely, the heterogeneity of assets earnings and the unequal access that individual have to them. Our paper point out the importance of non homogeneous assets returns upon pension reform effects on wealth distribution. We extend Matsuyama’s (2000) endogenous inequality model by allowing endogenous saving and investing behavior of agents facing different investment possibilities. In this paper, there is two assets yielding two different returns; an agent can lend his savings in the credit market, which earns him the equilibrium interest rate, or invest in an entrepreneurial fund which earns a constant high return. However the entrepreneurial fund requires a minimum investment.

We show that there is a unique wealth threshold dependent on the credit market interest rate, that separates the economics agents into poor-constrained agents, and rich-unconstrained agents. The poor-constrained agents are the lender that earn the equilibrium interest rate on their savings and the rich-unconstrained agents are the borrower earning the high net returns in the economy.

In this context we show that a higher contribution rate to the PAYG system increases the part of constrained-poor agents in the economy and decreases the interest rate. These effects are more important when the initial wealth distribution is less inequalitarian. The intuition behind is that an increase of the contribution rate causes a decrease in disposable wealth and savings of agents, since the contribution to the PAYG system is based on first period revenue. This affects more some middle class agents that become constrained. That is they lose the opportunity to invest in the high return asset in the economy. Since there

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are fewer unconstrained-rich agents, the decrease in capital demand is greater than the decrease in capital supply. This implies that the interest rate decreases to bring equilibrium in credit market.

Moreover, as the pension reform decreases the interest rate, it will generate a negative wealth effect for poor-constrained agents, and positive wealth effect for rich-unconstrained agents. The pension reform increases intragenerational inequality. Because the interest rate is the return rate of poor-constrained agents savings, they lose out from its decrease. Since the interest rate is the borrowing rate of rich-unconstrained agents, its decrease raises the net return of their savings. Therefore the rich-unconstrained agents benefit from the pension reform at the expense of poor-constrained agents.

Section 2 sets out the model and outlines its’ main properties. Section 3 establishes the conditions in which short run equilibrium with inequality exist. Section 4 develops the long run dynamics of wealth distribution and steady state equilibrium. Section 5 discusses the effect of a pension reform on interest rate, inequality, and agents wealth.

2 THE MODEL

The framework is a two-period OLG model with bequests. Each agent is a member of an infinitely-lived family. The only source of heterogeneity within a group of same-aged individuals is their inheritance. In their youth, every agent receives a bequest \((z_t)\) from their parents, and a wage endowment \((w_t)\). They leave a bequest \((z_{t+1})\) to their offspring in their second period of life. The agent contributes to the pension system at the rate \(\theta\), and receives a pension \((p_{t+1})\) when old.

During their youth, an agent allocates their income ( net of the contribution to the pension system) between consumption \((c_t)\), and savings \((s_t)\). He then has to decide how to invest his savings. He may invest his savings in the competitive credit market (become a lender), which earns a (gross) return of \(r_{t+1}\), or he can start an entrepreneurial project which yields a gross return of \(\rho\) per unit invested. However, starting an entrepreneurial project requires a minimum amount of capital, \(\kappa_t\). One can interpret the entrepreneurial project as an investment fund with minimum investment required, and yielding a high return.

The technology of the entrepreneurial project is defined by:

\[
F(k) = \begin{cases} 
0 & \text{if } k < \kappa \\
\rho k & \text{if } k \geq \kappa 
\end{cases}
\]

The minimum investment required \((\kappa_t)\) satisfies \(w_t(1-\theta) < \kappa_t\) at any time. It is optimal to invest the maximum possible above \(\kappa_t\), since the technology is of constant return. To invest at a level higher than their savings, the entrepreneur becomes a borrower of the funds supplied by the lender.

Following Matsuyama (2000) we assume that there is a moral hazard problem in the credit market. An agent who borrows an amount \((b_t)\) would refuse to honor its payment obligation \(r_{t+1}b_t\), if it were greater than the cost of default. We assume that the lender can, at most, seize a fraction \(\lambda \in (0,1)\) of the project output \((\rho k_t)\) in case of default. Knowing this, the lender would allow the entrepreneur to borrow only up to \(b_t = \lambda \rho k_t/r_{t+1}\). This defines the borrowing limit in the credit market caused by the hazard moral problem. As in Banerjee and Newman (1993) it is the possibility of default that urges the lender to limit the amount of their loan. This prevents any default in equilibrium. Thus, any agent who wants to invest \(k_t > \kappa_t\) must
have savings \( s_t = (1 - \lambda \rho / r_{t+1}) k_t \), which serve as a down payment\(^4\) for the loan. Those with bigger savings may undertake bigger entrepreneurial projects.

But, since there is a minimum requirement for setting up a project, only those with savings \( s_t \geq (1 - \lambda \rho / r_{t+1}) \kappa_t \) could effectively set-up a project. This defines the minimum level of savings required as down payment for borrowing and investing in a single project. Denote \( s^* = (1 - \lambda \rho / r_{t+1}) \kappa_t \).

The return on savings invested in the entrepreneurial project net of borrowing costs, is \( R_{t+1} = (1 - \lambda) \rho / (1 - \lambda \rho / r_{t+1}) \), which is higher than \( r_{t+1} \) until \( r_{t+1} < \rho \), and equal to \( r_{t+1} \) when \( r_{t+1} = \rho \). This implies that setting up an entrepreneurial project is always preferable when \( r_{t+1} < \rho \). The optimal strategy is to use all the savings as collateral to borrow, and invest the maximum compatible with the borrowing constraint, since the return on the project (net of the borrowing costs) is an increasing function of the capital invested \( k_t \).

Each agent chooses the optimal saving that maximizes his inter-temporal value function. Since the return on savings is higher when it is invested in the entrepreneurial project, some agents gain from forcing their savings to reach the entrepreneurial threshold. They will do so until it raises their value function. This is very similar to the optimal investment decisions of Galor and Zeira’s (1993) model. But in their model the interest rate is exogenous and constant. As in their model, inherited wealth determines investment decisions.

The agents program is summarized by:

\[
\max_{c_t, d_{t+1}, z_{t+1}} c_t^{1-\gamma} \left[ d_{t+1}^{\sigma} z_{t+1}^{\gamma} \right] \quad \text{s.t.} \quad \begin{cases} 
    c_t + s_t \leq w (1 - \theta) + z_t \\
    d_{t+1} + (1 + n) z_{t+1} \leq \hat{R}_{t+1} s_t + p_{t+1} \\
    \hat{R}_{t+1} = \begin{cases} 
        r_{t+1} & \text{if } s_t < s_t^* \\
        R_{t+1} & \text{if } s_t \geq s_t^*
    \end{cases}
\end{cases}
\]

where \( \hat{R}_{t+1} \) is equal to \( r_{t+1} \) if the agent is a lender and equal to \( R_{t+1} \) when the agent is an entrepreneur. Note that \( 0 < \gamma, \sigma < 1 \) and, \( d_{t+1} \) is the second period consumption. For a lender or an entrepreneur the optimal interior solution to his problem is:

\[
c_t = (1 - \gamma) \left[ w (1 - \theta) + z_t + \frac{p}{\hat{R}_{t+1}} \right] \quad \text{(4)}
\]

\[
d_{t+1} = (1 - \sigma) \gamma \hat{R}_{t+1} \left[ w (1 - \theta) + z_t + \frac{p}{\hat{R}_{t+1}} \right] \quad \text{(5)}
\]

\[
z_{t+1} = \frac{\sigma}{1 + n} \gamma \hat{R}_{t+1} \left[ w (1 - \theta) + z_t + \frac{p}{\hat{R}_{t+1}} \right] \quad \text{(6)}
\]

\[
s_t = \gamma (w (1 - \theta) + z_t) - (1 - \gamma) \frac{p}{\hat{R}_{t+1}} \quad \text{(7)}
\]

One can easily compute the corresponding value function of a lender \( \hat{R}_{t+1} = r_{t+1} \) or an entrepreneur \( \hat{R}_{t+1} = R_{t+1} \) as:

\[
V \left( \hat{R}_{t+1}, z_t \right) = (1 + n)^{-\sigma \gamma} (1 - \gamma)^{1-\gamma} (1 - \sigma)^{1-\sigma \gamma} \sigma^\gamma \gamma^\gamma \left( \hat{R}_{t+1} \right)^\gamma \left[ w (1 - \theta) + z_t + \frac{p}{\hat{R}_{t+1}} \right] \quad \text{(8)}
\]

\(^4\)We exclude the possibility that second period pensions benefits enter in the ressources that the lender could seize in case of default for simplicity.
One can compute the value function of an agent who saves exactly the borrowing limit to set up one project. It is the optimal value function of the corner solution of the borrower problem:

\[ V(R_{t+1}, z_t) = (1 + n)^{-\sigma \gamma} \left( \frac{\gamma}{1 - \gamma} \right)^\gamma (1 - \sigma)^{(1 - \gamma) \sigma \gamma} (R_{t+1})^\gamma [w(1 - \theta) + z_t - s]. \]  

Since there’s a threshold saving to set up one project, it may be value function enhancing to over-save to reach this borrowing limit. This means that some agents will force their first period savings to reach this borrowing limit, and attain the threshold to become entrepreneurs. Since the inheritance is the only source of heterogeneity between agents, one can derive an inheritance threshold that separates the population into two categories. The constrained agents, for whom it’s optimal to become a lender, and the unconstrained borrower-entrepreneur. Equalizing the respective value function of a lender and of a borrower setting-up one project, i.e. \( V(r_{t+1}, z_t) = V(R_{t+1}, z_t) \), yields the inheritance threshold. The inheritance threshold is the inheritance of the agent be he borrower or lender in terms of value function. It writes:

\[ z_t = \frac{(1 - \gamma) (r_{t+1})^\gamma \frac{w}{r_{t+1}} + (R_{t+1})^\gamma s}{(R_{t+1})^\gamma - (1 - \gamma) (r_{t+1})^\gamma} - w(1 - \theta). \]  

Thus, the household whose inheritance is above this level is rich enough to borrow and become an entrepreneur. They will use all their savings as a down payment to borrow the maximum consistent with the borrowing limit. The households whose inheritance is below this level, are not rich enough to attain to the borrowing limit, and have no interest in stretching their savings to reach the borrowing limit.

Since \( R_{t+1} \) is a function of \( r_{t+1} \), the inheritance threshold is both a single valued function and an increasing function of the interest rate. When the interest rate is high, borrowing is costly and therefore the down payment required is higher. As saving is an increasing function of the inheritance, the corresponding inheritance threshold is higher.

The wealth threshold is therefore endogenously determined by the evolution of the interest rate. Thus, being rich or poor, is a relative position for an endogenous wealth threshold. In the next section, we complete the model by describing the credit market equilibrium condition and the macroeconomic equilibrium.

### 3 INHERITANCE DISTRIBUTION AND SHORT-RUN EQUILIBRIUM.

Let \( G_t(z) \) denote the inheritance distribution function across households, defined in \( \mathbb{R}_+ \) between \( z_{\text{min}} \) and \( z_{\text{max}} \) with values in \((0, 1)\). The pension system is a fully funded “pay as you go” (PAYG) pension scheme. Pensions are financed by a proportional tax on the first period wage. The budgetary equilibrium of the pension system is written:

\[ N_t \theta w_t = N_{t-1} p_t \]  

\( N_t \) denotes the number of youth in \( t \), and \( N_{t-1} \) the number of old people. The pension system is "Bismarkian" since no direct redistribution is intended. Assume that the wage is constant. Let \( p_t \equiv \tau_t w_{t-1} \), where \( \tau_t \)

\(^5\)Note that \( sw(1 - \theta) < (1 - \lambda p/r) \kappa \), otherwise no individual would ever be constrained.
designates the replacement rate. Thus one can easily show that \( \tau_t = (1 + n) \theta \) with \( n \) being the constant growth rate of the population.

The equilibrium interest rate is determined by the credit market equilibrium condition. The aggregate net demand of capital by entrepreneurs must be equal to the aggregate net supply of capital by lenders. It can be shown that the equilibrium interest rate must agree with the following condition\(^6\):

\[
\lambda \rho < r_{t+1} \leq \rho
\]

That is there are two types of equilibrium. Firstly, \( \lambda \rho < r_{t+1} < \rho \), and secondly, \( r_{t+1} = \rho \). In the first equilibrium, the projects’ return is always superior to the interest rate (which is the return for a lender). In the second equilibrium, the equilibrium interest rate is equal to the project returns, meaning that all agents earn the same return on their savings. The first type of equilibrium yields inequality in earnings, while the second type of equilibrium leads to equality in earnings. Since we are interested in the joint evolution of both the interest rate, and wealth distribution with inequality, we will focus the analysis on the case where equilibrium interest rate is strictly lower than the project return.

In the case \( \lambda \rho < r_{t+1} < \rho \), the aggregate net demand of capital for investment is the demand of capital backed by their savings of all agents whose inheritance is above the inheritance threshold:

\[
N_t \int_{z_t}^{z_t} \frac{\alpha - \lambda \rho}{r_{t+1}} s(z, R_{t+1}) dG_t(z)
\]

where \( s(z, R_{t+1}) \) designates the savings of entrepreneurs. Aggregate net supply of capital is the aggregate saving of all constrained agents. That is all agents whose inheritance is below the inheritance threshold:

\[
N_t \int_{z_t}^{z_t} s(z, r_{t+1}) dG_t(z)
\]

where \( s(z, r_{t+1}) \) designates the savings of lenders or constrained agents. The credit market equilibrium condition is thus:

\[
\frac{\lambda \rho}{r_{t+1}} \left[ 1 - G_t(z) \right] \left[ \gamma \left( w (1 - \theta) + E_t[z \mid z \geq z_t] \right) - (1 - \gamma) \frac{p}{R_t+1} \right] = \gamma \left( w (1 - \theta) + E_t[z \mid z \leq z_t] \right) - (1 - \gamma) \frac{p}{R_t+1}.
\]

This mechanism is very close to that of Aghion and Bolton (1997). In their model middle class borrowers express the demand of capital, while poor and rich lenders express the supply of capital. It is the uncertainty of the return on the entrepreneurial project that limits the demand for capital in their model. In our model the constant return on each unit of capital invested makes the demand for capital increase with the agents wealth. To achieve the assessment of the model, let us establish the existence and the uniqueness of the short-run equilibrium.

**Proposition 1** Given \( G_t(z) \), there exists a temporary equilibrium with inequality, i.e. \( \{r_{t+1}, z_t\} \) with \( r_{t+1} \in (\lambda \rho, \rho) \).

\(^6\)see MATSUYAMA (2000)
Proof. The system equation that defines the temporary equilibrium, is a function of \( r_{t+1} \) and \( z_t \). Since \( z_t \) is a function of \( r_{t+1} \), then the equilibrium exists and is unique if, for any equilibrium interest rate \( \lambda \rho < r_{t+1} < \rho \), there exists a unique \( z_t \). Let write \( z_t \) as a function of \( r_{t+1} \) by equation (10) so that \( z_t \equiv \Psi (r_{t+1}) \). One can rewrite the credit market equilibrium condition as a function of \( r_{t+1} \), and \( z_t \) as \( \Phi (z_t, r_{t+1}) = 0 \). One can show that \( \Phi (\Psi (r_{t+1}), r_{t+1}) \) is monotonic and decreasing\(^7\) with \( r_{t+1} \). Since \( \lim_{r_{t+1} \to \lambda \rho} \Phi (\Psi (r_{t+1}), r_{t+1}) = +\infty \) and \( \lim_{r_{t+1} \to \rho} \Phi (\Psi (r_{t+1}), r_{t+1}) < 0 \), then \( r_{t+1} \) exists and is unique. Since \( \Psi (r_{t+1}) \) is monotonic and increasing then for any equilibrium \( r_{t+1} \), \( z_t \) is well defined and unique. \( \blacksquare \)

The distribution of inheritance determines the equilibrium interest rate, and therefore the equilibrium inheritance threshold which divides the population into two categories. The rich borrowers-entrepreneurs and the poor lenders. Thus, the inheritance distribution has a strong effect on the macroeconomic equilibrium.

4 DYNAMICS OF WEALTH AND LONG-RUN EQUILIBRIUM.

The distribution of wealth not only determines equilibrium in period \( t \), but also determines the next period’s distribution of inheritances \( G_{t+1} \). Let \( \phi = \sigma \gamma / (1 + n) \). Then the dynamic is written:

\[
  z_{t+1} = \begin{cases} 
    1 - \theta + \frac{(1+n)\theta}{r_{t+1}} & \phi r_{t+1} w + \phi r_{t+1} z_t \text{ if } z_t < z_t \\
    1 - \theta + \frac{(1+n)\theta}{r_{t+1}} & \phi R_{t+1} w + \phi R_{t+1} z_t \text{ if } z_t \leq z_t 
  \end{cases}
\]

(13)

where the two returns are given by the credit market equilibrium. Equation (13) describes the relationship dynamic between inheritance and bequest for lenders and entrepreneurs.

By definition \( \phi r_{t+1} < 1 \) as \( \phi R_{t+1} \) is inferior to 1. Let us examine the long term behavior of this economy. If the difference equations governing the wealth transitions are stable, it would be easy to prove the existence of a stationary wealth distribution. However, the fact that these difference equations depend on the interest rate raises the possibility that the process may not be stable. The concern here is that the interest rate may change continually. The following arguments rule out\(^8\) this possibility. The idea is to show that the interest rate varies during a finite number of periods to reach its long term value. Once the interest rate has reached its long term value, agents inheritances will converge to their long term value.

What drives the movement of the interest rate is the movement of agents being constrained and unconstrained. This involves a change in the relative aggregate demand and supply of capital. Once the groups of constrained and unconstrained agents are stable, the interest rate will quickly stabilize. We show that it is only during few periods that agents could change of side that is being constrained and unconstrained and therefore the two groups will be stable and therefore the interest rate. The following figure illustrates this property of the joint dynamics of wealth threshold and interest rate.

\(^7\) Calculus details and graphic illustration are available on request.

\(^8\) The argument is the same as in Ghatak and Jiang (2001).
Note that the difference equations (13) are order-preserving. That is, \( z_{i,t+1} > z_{j,t+1} \) if and only if \( z_{i,t} > z_{j,t} \). At any initial date \( t \), given \( G_t(z) \) there exists \( r_{t+1} \) and \( z_{t+1} \). It is the first period interest rate and inheritance threshold which separates the population into two categories. The rich earning \( R_{t+1} \) on their savings and the poor earning \( r_{t+1} \) on their savings. This difference on earnings magnifies intragenerational inequality. That is the gap between the old rich and old poor is bigger than when they were all young. Another effect is that the rich agents climb further away from the inheritance threshold. This means that there exists a gap between the "richest poor" and the "poorest rich" in the first generation. Since inheritance is order preserving, the gap will not disappear until the interest rate is not stable, and the agents are not earning the same return. So inequality in the next generation is greater, since inheritance transmission is order preserving. Since the heirs of rich agents are richer, compared to heirs of poor agents, their demand of capital (during the second period) is relatively greater. This wealth effect drives the interest rate up, but a joint effect is that the inheritance threshold is greater. This implies that some rich heirs could become constrained, and raises the aggregate supply of capital whilst driving the interest rate down. This is the threshold effect. The interest rate which results from this may be greater or lower than the first period interest rate, depending on the initial inheritance distribution. Assume that the wealth effect causes\(^9\) the interest rate’s moving, then the interest rate goes up. In this case some rich agents could topple over into the constrained side. But it is not enough to offset the increase of capital demand. Therefore the second period interest rate is greater than the first periods’, meaning that intragenerational inequality drops. The fact that wealth effect and threshold effect are opposite, limits the magnitude of the change of the interest rate and therefore limits the effective number of agents who change sides. But now there are two categories of second period poor agents. The heirs of first period poor, who stay poor, and the heirs of some first period rich agents who become constrained by the threshold effect (i.e. some middle classes). These two categories of poor agents earn the same return, so the gap between them is preserved. But now they all benefit from a higher interest rate while the heirs of rich agents who stay rich (that is rich agents of second period) earn the highest return in the economy. These rich agents climb away from the period inheritance threshold. The cumulative effect of remaining rich in both periods magnifies the gap between their wealth and the period inheritance threshold. After that, transmission of inheritances take place. Since third generation poor youth

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\(^9\)It is the converse if the threshold effect carry the interest rate moving.
are relatively richer and more numerous, their supply of capital is relatively greater. This tends to decrease
the interest rate and therefore the inheritance threshold. It is the wealth effect which now implies a decrease
in the interest rate. A decrease of the inheritance threshold implies that some heirs of middle class parents
could become unconstrained and this would increase capital demand. It is the threshold effect which now
implies an increase in the interest rate. Since the magnitude of the variation of the interest rate is low, only
the middle class agents could change side. The wealth effect will determine that the interest rates’ movement
will be downwards. Young rich agents of the third period will benefit from this decrease in the interest rate
only when they’re old. This makes them climb away from the period inheritance threshold. Even if in the
following period the interest rate goes up only very few of them could fall into being constrained. Thus, it is
only during few periods that agents change sides. Therefore when the two groups are constant, the interest
rate will quickly reach its long term value. The following lemma summarizes this property of stability and
convergence of the interest rate dynamic.

**Lemma 2** *The interest rate varies during a finite number of period to reach its long term value.*

Once the interest rate has reached its stationary value, the convergence processes of the wealth dynamic
start. As illustrated in figure 2 below, individuals who inherit less than $z_{n\infty}$ are constrained lenders, as are
their descendants in all future generations. In the long-run, constrained agents inheritances converge to
$z_{n\infty}^c < z_{n\infty}:$

$$z_{n\infty}^c = \frac{\phi r_{n\infty} w (1-\theta) + \phi p}{1 - \phi r_{n\infty}}$$  \hspace{1cm} (14)

Individuals who inherit more than $z_{n\infty}$ invest in entrepreneurial projects, as do their offspring throughout all
generations. Their bequests converge to $z_{n\infty}^u \geq z_{n\infty}:$

$$z_{n\infty}^u = \frac{\phi \left(\frac{(1-\lambda) r_{n\infty}}{r_{n\infty} - \lambda p}\right) w (1-\theta) + \phi p}{1 - \phi \left(\frac{(1-\lambda) r_{n\infty}}{r_{n\infty} - \lambda p}\right)}$$  \hspace{1cm} (15)

Illustration of the convergence processus by figure 2:
As seen in equation (15), the wealth of the unconstrained converges only when the steady-state interest rate satisfies \( r_\infty > \lambda \rho / (1 - \phi (1 - \lambda) \rho) \). The long-run inheritance threshold is given by the steady state interest rate and the minimum investment. It is written:

\[
\hat{z}_\infty = \frac{(1 - \gamma) \rho}{r_\infty} + ((1 - \lambda) \rho)^\gamma \frac{1}{r_\infty} (r_\infty - \lambda \rho)^{1 - \gamma} - w (1 - \theta)
\]

A necessary condition of existence of \( \hat{z}_\infty \) is \( r_\infty < \left[ \lambda \rho + (1 - \lambda) \rho / (1 - \gamma) \right]^{1/\gamma} \) which is verified.

**Proposition 3** In a stationary state with inequality, \( r_\infty \in I \equiv \left( \frac{\lambda \rho}{1 - \phi (1 - \lambda) \rho}, \rho \right) \) and individual wealth is equal to either \( z^{c}_{\infty} = (\phi r_\infty w (1 - \theta) + \phi p) / (1 - \phi r_\infty) \) or \( z^{u}_{\infty} = \left[ \phi \left( \frac{(1 - \lambda) \rho r_\infty}{r_\infty - \lambda \rho} \right) w (1 - \theta) + \phi p \right] / (1 - \phi (1 - \lambda) \rho r_\infty) \).

Note that the interval \( r_\infty \in I \equiv \left( \frac{\lambda \rho}{1 - \phi (1 - \lambda) \rho}, \rho \right) \) reduces to \( r_\infty = \rho \) if \( \lambda = 1 \). Thus, the lemma also implies that \( \lambda < 1 \) is a necessary condition for the existence of a steady state of inequality. Defining a steady state as a state in which the level of inheritance is stationary for all households, and the size of the two categories (constrained and unconstrained) is stable. Denote \( s^{u}_{\infty} (r_\infty, z^{u}_{\infty}) \) as the steady state unconstrained savings and \( s^{c}_{\infty} (r_\infty, z^{c}_{\infty}) \) as the steady state constrained agents savings. The steady state credit market equilibrium condition is written,

\[
\alpha_\infty s^{u}_{\infty} (r_\infty, z^{u}_{\infty}) = (1 - \alpha_\infty) \left( \frac{\lambda \rho}{1 - \lambda \rho} \right) s^{u}_{\infty} (r_\infty, z^{u}_{\infty})
\]

where \( \alpha_\infty \equiv G_\infty (z^{c}_{\infty}) < 1 \) is the steady state fraction of constrained households. From this condition one can express easily \( \alpha_\infty \) as a function of \( r_\infty \):

\[
\alpha_\infty (r_\infty) = \frac{\lambda \rho s^{u}_{\infty} (r_\infty, z^{u}_{\infty})}{(r_\infty - \lambda \rho) s^{c}_{\infty} (r_\infty, z^{c}_{\infty}) + \lambda \rho s^{u}_{\infty} (r_\infty, z^{u}_{\infty})}
\]

One can easily show that \( \alpha_\infty \) is decreasing in \( r_\infty \in I \), and is in the range \((\lambda, 1)\). This suggests that for any steady state interest rate, \( r_\infty \in I \), one can always find a unique value of \( \alpha_\infty \in (\lambda, 1) \) that satisfies equation (18). Thus, to demonstrate the existence of a two-point steady state, it suffices to check the inequalities in the following condition:

\[
z^{c}_{\infty} (r_\infty) < \hat{z}_\infty (r_\infty) \leq z^{u}_{\infty} (r_\infty)
\]

To summarize the solution, let’s distinguish between two cases. In the first one \( \hat{z}_\infty (r_\infty) \) intersect with the two curves \( z^{c}_{\infty} (r_\infty) \) and \( z^{u}_{\infty} (r_\infty) \) in the interval \((\lambda \rho, \rho)\), and, in the second one it intersects only with \( z^{c}_{\infty} (r_\infty) \) in this interval. In the first case, denote \( r^\gamma \) the solution of \( z^{c}_{\infty} (r_\infty) = \hat{z}_\infty (r_\infty) \), and \( r^\gamma \) the solution of \( z^{c}_{\infty} (r_\infty) = z^{u}_{\infty} (r_\infty) \). Then there exists a continuum of steady state with \( r_\infty \in (r^-, r^+) \) where \( r^- \equiv \max \left\{ \frac{\lambda \rho}{1 - \phi (1 - \lambda) \rho}, r^\gamma \right\} \) and \( \alpha_\infty = \alpha_\infty (r_\infty) \in (\alpha (r^+), \alpha (r^-)) \). In the second case, denote \( r^\gamma \) and \( r^+ \) the two solutions of \( z^{c}_{\infty} (r_\infty) = \hat{z}_\infty (r_\infty) \). Then there exists a continuum of steady state with \( r_\infty \in (r^-, r^+) \) where \( r^- \equiv \max \left\{ \frac{\lambda \rho}{1 - \phi (1 - \lambda) \rho}, r^\gamma \right\} \) and \( \alpha_\infty = \alpha_\infty (r_\infty) \in (\alpha (r^+), \alpha (r^-)) \).

Figure 3 illustrates one of the four generic ways those curves may intersect.\(^{10}\)

\(^{10}\)We find the four cases of intersection between the curves line up with those of MATSUYAMA (2000). So we represent only one of them.
In figure 3, $r^- = r_{\text{inf}}$ is greater than $r^* = \frac{\lambda \rho}{1 - \theta(1 - \lambda) \rho}$ and $r^+$ is the solution of $z_{\infty} (r_{\infty}) = z^u_{\infty} (r_{\infty})$. Any $r_{\infty} \in (r^-, r^+)$ corresponds to a steady state with inequality characterized by a two-point distribution of wealth. The degree of inequality differs across the steady states. A high interest rate is associated with lesser inequality, both in terms of relative groups and wealth. Indeed a higher interest rate implies that $z^u_{\infty} (r_{\infty})$ is smaller (the rich are less rich), that $z^c_{\infty} (r_{\infty})$ is greater (the poor are richer) and that $\alpha_{\infty} (r_{\infty})$ is smaller (the fraction of constrained agents is smaller).

5 Pension reform and wealth distribution.

In this section we study how a higher contribution rate to the pension system affects equilibrium interest rate and inequality. It modifies both individual and aggregate savings, and therefore aggregate supply and demand of capital. This will in turn affect the interest rate and the inheritance threshold between constrained and unconstrained agents.

From equation (10), one can see that $z_{t}$ is a function of $r_{t+1}$ and $\theta$. To determine the effect of a reform of the pension system ($d\theta > 0$) on the wealth threshold, we must differentiate $z_{t} (r_{t+1}, \theta)$. We get

$$dz_t = \frac{\partial z_t}{\partial \theta} d\theta + \frac{\partial z_t}{\partial r_{t+1}} dr_{t+1}$$

(20)

One can easily see that both $\frac{\partial z_t}{\partial \theta}$ and $\frac{\partial z_t}{\partial r_{t+1}}$ are positive. The effect on the credit market equilibrium condition (CMEC) yields the variation of $r_{t+1}$. From equation (12) rewrite the CMEC as $\Phi (z_t, r_{t+1}, \theta) = 0$. Differentiating this equation and substituting equation (20) yields

$$dr_{t+1} = -\frac{\left( \frac{\partial \Phi (z_t, r_{t+1}, \theta)}{\partial \theta} + \frac{\partial \Phi (z_t, r_{t+1}, \theta)}{\partial z_t} \frac{\partial z_t}{\partial r_{t+1}} \frac{dz_t}{d\theta} \frac{\partial z_t}{\partial \theta} \right)}{\left( \frac{\partial \Phi (z_t, r_{t+1}, \theta)}{\partial z_t} \frac{\partial z_t}{\partial r_{t+1}} + \frac{\partial \Phi (z_t, r_{t+1}, \theta)}{\partial r_{t+1}} \right)} d\theta < 0$$

(21)

For all calculus in this section see the appendix.
An increase in the contribution rate to the pension system leads to a decrease of the equilibrium interest rate. As seen in equation (20) we have two opposite effects on the inheritance threshold. Increasing $\theta$ raises the inheritance threshold while the involved decrease of the interest rate tends to decrease the inheritance threshold. The intuition behind this result is simple. The first effect of a rise of the contribution rate $\theta$ is to increase the wealth threshold since it decreases the disposable wealth and the savings of all agents. Indeed, the contribution is made on first period revenue and therefore the disposable wealth to allocate between consumption and savings drops. Since agents savings drops the borrowing threshold is relatively higher. It is a quantity effect on the inheritance threshold. Thus, some agents who were unconstrained become constrained. This increase in the contribution rate affects more some middle class agents near the borrowing threshold, since they fall in the constrained side after the reform. This modifies the equilibrium in the credit market by decreasing both the supply and demand of capital, but the demand of capital decreases relatively more since the unconstrained agents become fewer. This implies that the interest rate decreases to balance the credit market. A decrease in the interest rate makes the down payment required to borrow lower. This leads to a decrease of the borrowing limit and therefore to a decrease of the inheritance threshold. This is a price effect on the inheritance threshold yielded by the variation of the interest rate. But this price effect is not enough to offset the initial increase of the wealth threshold due to the increase of $\theta$. Thus an increase of the contribution rate to the pension system leads to an increase of the inheritance threshold, that is more poor-constrained agents, and to a decrease of the interest rate. The importance of this effect of the pension reform depends on the initial wealth distribution.

**How wealth distribution affects pension reform.**

The degree of inequality and dispersion in the initial wealth distribution determine the magnitude of the effect of the pension reform. Indeed increasing the contribution rate to the pension system leads to an increase of the inheritance threshold. But the magnitude of this increase depends on the degree of inequality in the wealth distribution. Whether many agents are falling down the constrained side or not and the part of the aggregate wealth they hold determine the importance of the interest rate variation. To illustrate how the wealth distribution affect the effect of the pension reform on the inheritance threshold and on the interest rate, we simulate the model. The following table summarize the evolution of $Z$ which express the percentage of constrained agents in the economy while $r_{t+1}$ is the interest rate in percentage. $I_{gini}$ is the Gini index of the initial distribution, it is an index of inequality and of degree of wealth concentration.

<table>
<thead>
<tr>
<th>$I_{gini}$</th>
<th>$Z$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.31</td>
<td>84.2</td>
<td>3.97</td>
</tr>
<tr>
<td>0.51</td>
<td>86.4</td>
<td>3.90</td>
</tr>
<tr>
<td>0.70</td>
<td>88</td>
<td>3.86</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\theta = 20%$</th>
<th>$Z$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>84.2</td>
<td>3.97</td>
<td>3.90</td>
</tr>
<tr>
<td>86.4</td>
<td>3.90</td>
<td>3.86</td>
</tr>
<tr>
<td>88</td>
<td>3.86</td>
<td>3.81</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\theta = 35%$</th>
<th>$Z$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>87.7</td>
<td>3.88</td>
<td>3.84</td>
</tr>
<tr>
<td>89.1</td>
<td>3.84</td>
<td>3.81</td>
</tr>
<tr>
<td>90.4</td>
<td>3.81</td>
<td></td>
</tr>
</tbody>
</table>

Table: Pension reform and wealth distribution.

---

We calibrate the model to get equilibrium with inequality. The parameters are $\lambda = 0.3$, $\gamma = 0.55$, $\sigma = 0.4$, $n = 1.5\%$, $\rho = 6\%$ and a level of minimum investment consistent with the distribution. The distribution is a log-normal random draw. We use Matlab to simulate the model.
For an initial distribution with Gini index of 0.31 that is few inequality and for a contribution rate to the pension system of 20% on wage, the model generates 84.2% of constrained agents and an interest rate of 3.97%. When the Gini index of the initial distribution is 0.70 that is a high degree of inequality, then, for the same contribution rate of 20%, the model generates a higher degree of constrained agents 88% but a lower interest rate 3.86%. The intuition behind is easy to grasp. A high Gini index means that the rich agents in the economy hold a higher part of the aggregate wealth compared to an economy with a low Gini index. Thus the rich 12% of the economy with high initial economy hold a higher part of the aggregate wealth than the 15.8% rich agents of the economy with low initial economy. Therefore, as their demand of capital, with the lever effect, is much more important than that of rich agents of the low inequality economy, the interest rate is lower.

Increasing the contribution rate to the pension system from 20% to 35% increases the percentage of constrained agents from 84.2% to 87.7% (+ 3.5%) and decreases the interest rate from 3.97% to 3.88% (-0.09%) when there is few inequality in the initial distribution. Thus an increase of the contribution rate to the pension system increase inequality in the economy. But the variation of the number of constrained agents and of the interest is less important when the initial inequality is high. Indeed, when initial Gini index is 0.70, increasing \( \theta \) causes a variation of constrained-poor agents of +2.4% and a decrease of the interest rate of -0.05%. Therefore the effect of the pension reform is less important when the initial inequality is high. Indeed, when the initial Gini index is low there may be more agents around the wealth threshold. In this case, when the wealth threshold increases with \( \theta \) many agents fall down into the constrained side what induces that the interest rate has to decrease more to balance the credit market. While when the initial inequality is high there may be less agents around the wealth threshold. In this case, when the wealth threshold increase with \( \theta \) less agents are involved and that is less agents become constrained. In this case, the variation of the interest has to be less important to balance the credit market.

**Wealth effect of a reform of the pension system.**

The variation of the interest rate has asymmetric effect on the return of the different assets in the economy. Indeed the pension reform decreases the credit market equilibrium interest rate. A decrease of the interest rate increases the net return of investing in the fund while it decreases the return of lending in the credit market.

The effect on constrained agents’ wealth is given by:

\[
dz_c^\infty = \frac{\partial z_c^\infty}{\partial \theta} d\theta + \frac{\partial z_c^\infty}{\partial r^\infty} dr^\infty
\]

where \( \frac{\partial z_c^\infty}{\partial r^\infty} > 0 \) since the interest rate is the return on constrained agents’ investment, while the sign of \( \frac{\partial z_c^\infty}{\partial \theta} \) is ambiguous, depending on whether the return of the pension system \((1+n)\) is superior or inferior to the interest rate. If \( r^\infty > (1+n) \), then \( \frac{\partial z_c^\infty}{\partial \theta} \) is negative and therefore \( dz_c^\infty < 0 \). In this case the constrained agents lose twice-over. Firstly, the increase of the contribution rate to the obligatory pension system forces them to invest more in the retirement system (which earn less). Secondly, it decreases the interest rate which is the return on their private savings. If \( r^\infty < (1+n) \), then \( \frac{\partial z_c^\infty}{\partial \theta} \) is positive. The pension system earns more than lending in the credit market, so that the negative effect of the involved decrease of the interest rate is balanced by the positive effect of the increase of the pension. But as the part of their earnings from private
savings is greater than that one from pensions, the constrained-poor agents always lose out from reform.

The effect on the unconstrained agents wealth is given by:

\[
dz_u = ∂z_u/∂θ dθ + ∂z_u/∂r dr
\]

where \( ∂z_u/∂r < 0 \) since the interest rate is the borrowing rate of the unconstrained agents. \( ∂z_u/∂θ \) is negative since for the unconstrained agents the pension system always earns less than their investment. The direct effect of an increase of the contribution rate to the pension system is always negative for unconstrained agents. But this direct effect may be offset by the positive indirect effect due to the involved decrease in the interest rate. Indeed, since \( ∂z_u/∂r < 0 \), a decrease in the interest rate corresponds to a decrease in the borrowing rate of rich agents and therefore to an increase of their net return. Since the part of their wealth invested in the market is greater than the part devoted to the pension system, the unconstrained agents benefit from the pension reform because the involved decrease of the interest. This magnifies the conflict, between constrained and unconstrained agents, over a pension reform.

6 Concluding remarks.

There is a conventional view that in OLG framework PAYG system leads to less capital accumulation and high interest rate but is more inequality reducing than a privately funded system. PAYG pension system may reduce inequality at least in two ways. One is PAYG system reduces intragenerational inequality when the benefits from the system are not exactly proportional to the contributions of agents. Another is that when individual face risk in earnings or assets returns, the PAYG system by sharing risk across individuals moderates the transmission of individual risk into inequality (see Deaton, Gourinchas and Paxson (2000)).

By taking into account different assets returns and unequal access to them, we find that PAYG pension system generates lower level of interest rate and increases wealth inequality. Moreover, more the initial distribution is egalitarian more the effect of the pension reform is important, that is more the increase of the part of constrained-poor agents is high and the decrease of the interest rate is important. As the variation of the interest rate causes a variation of assets returns in the economy, it modifies agents net wealth gains. Since the credit market interest rate is the return of poor-lender assets, its decrease disfavor them, while it increases the net return of rich-borrower assets. Therefore unconstrained rich agents benefits from the reform at the expense of constrained poor agents.

References


APPENDIX

Pension Reform:

Derivatives of $z_t$

$$z_t = \frac{\left((1 - \gamma)(1 + n)\theta w_t + \frac{k}{(1 - \lambda^\gamma)(r_t + \lambda)^{1 - \gamma}}\right)}{(1 - \gamma)(1 + n)\theta w_t + \frac{k}{r_t + \lambda} - w_t (1 - \theta)}$$

$$\frac{\partial z_t}{\partial \theta} = \frac{(1 - \gamma)(1 + n)w_t}{r_t + \lambda (1 - \gamma)(1 + n)w_t + w_t} + w_t > 0$$

$$\frac{\partial z_t}{\partial r_{t+1}} = \frac{\partial}{\partial r_{t+1}} > 0$$

$$dz_t = \frac{\partial z_t}{\partial \theta} d\theta + \frac{\partial z_t}{\partial r_{t+1}} dr_{t+1}$$

The differential of the CME for $z$ is:

$$d\Phi(z, r_{t+1}, \theta) = \frac{\partial \Phi(z, r_{t+1}, \theta)}{\partial \theta} d\theta + \frac{\partial \Phi(z, r_{t+1}, \theta)}{\partial z} dz_t + \frac{\partial \Phi(z, r_{t+1}, \theta)}{\partial r_{t+1}} dr_{t+1} = 0$$

$$\frac{\partial \Phi(z, r_{t+1}, \theta)}{\partial z} dz_t = -\frac{\partial \Phi(z, r_{t+1}, \theta)}{\partial \theta} d\theta - \frac{\partial \Phi(z, r_{t+1}, \theta)}{\partial r_{t+1}} dr_{t+1}$$

Thus

$$\left(\begin{array}{c}
\frac{\partial \Phi(z, r_{t+1}, \theta)}{\partial z} dz_t \\
\frac{\partial \Phi(z, r_{t+1}, \theta)}{\partial r_{t+1}} dr_{t+1}
\end{array}\right) < 0$$

therefore

$$dz_t = \frac{\partial z_t}{\partial \theta} d\theta + \frac{\partial z_t}{\partial r_{t+1}} (\frac{\partial \Phi(z, r_{t+1}, \theta)}{\partial \theta} + \frac{\partial \Phi(z, r_{t+1}, \theta)}{\partial r_{t+1}}) d\theta$$

Individual wealth effect:

For unconstrained agents

$$z_{u, \infty} = \frac{\phi \left(1 - \lambda \rho_{\infty} w_t \right) (1 - \theta) + \phi (1 + n) w_t}{1 - \phi \frac{(1 - \lambda \rho_{\infty} w_t)}{r_{\infty} - \lambda}}$$

$$\frac{\partial z_{u, \infty}}{\partial \theta} = \frac{\phi \left(1 - \lambda \rho_{\infty} w_t \right) (1 + n) \frac{w_t}{1 - \phi \frac{(1 - \lambda \rho_{\infty} w_t)}{r_{\infty} - \lambda}}}{1 - \phi \frac{(1 - \lambda \rho_{\infty} w_t)}{r_{\infty} - \lambda}} < 0$$

If we assume that $(1 + n) < R_{\infty}$

For constrained agents:

$$z_{c, \infty} = \frac{\phi \left(1 - \lambda \rho_{\infty} w_t \right) (1 - \theta) + \phi (1 + n) w_t}{1 - \phi \frac{(1 - \lambda \rho_{\infty} w_t)}{r_{\infty} - \lambda}}$$

$$\frac{\partial z_{c, \infty}}{\partial \theta} = \frac{\phi \left(1 - \lambda \rho_{\infty} w_t \right) (1 + n) \frac{w_t}{1 - \phi \frac{(1 - \lambda \rho_{\infty} w_t)}{r_{\infty} - \lambda}}}{1 - \phi \frac{(1 - \lambda \rho_{\infty} w_t)}{r_{\infty} - \lambda}} < 0$$

If $(1 + n) < R_{\infty}$

$$> 0$$