Productivity Slowdown and Resurgence: The Role of Capital Obsolescence

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In a recent work, Karl Whelan [2003] argues that the hypothesis of balanced growth is firmly rejected by postwar U.S. data. There is some clear evidence that the ratio of real investment to real consumption has exhibited an upward trend since the late 1950s. In this case, the traditional one-sector model of economic growth provides a poor description of the long-run behavior of the U.S. economy. In this paper, I develop a simple two-sector model of economic growth in which the obsolescence of capital goods is endogenous. Numerical simulations of unbalanced growth paths suggest that the rapid decline in the relative price of equipment goods observed since the late 1950s in the U.S. has shortened the average service-life of equipment, which, in turns, induced a long-lasting underestimation of the growth rate of Total Factor Productivity.

Classification JEL : O33, O41, O47.

INTRODUCTION

Stylized facts characterizing the post-war dynamics of industrialized economies are now well known. The United States and European economies have enjoyed striking high rates of productivity and economic growth until the late sixties. Then, the growth rates of labor productivity and of total factor productivity (TFP) decelerated sharply both in the U.S. and in most other industrialized countries. This persistent productivity slowdown is puzzling and one must recognize that neither growth accounting literature nor modern growth theories have provided a satisfactory explanation. Rejecting the idea that technological progress has been dormant during this period, many observers argued that the so-call productivity slowdown was, in reality, a statistical fiction. More specifically, they suggest that some or all of the measured productivity slowdown is accounted for by increased mismeasurement of output since the mid-seventies. This measurement hypothesis is appealing because evidence for mismeasurement is persuasive. Unfortunately, the hypothesis that mismeasurement is worth after the productivity slowdown than before is not supported by recent empirical studies (cf. Sichel [1997] and Triplet [2002]). This statement leaves the productivity slowdown largely unexplained. But

recently, macroeconomic performances took different directions in Europe and in the U.S. While no sign of productivity revival could be detected in Europe, researchers became aware that the second half of the 1990s was marked by a resurgence of productivity growth in the US. Attention quite naturally turned from analysis of the post-1973 slowdown to the post-1995 acceleration of productivity.

This paper proposes a new measurement explanation compatible with both the slowdown and the resurgence of productivity. Unlike the studies previously cited, the focus is put on mismeasurement of inputs, not outputs. The main idea is to consider seriously the results of empirical and theoretical studies that claim that lifespan of capital goods is not exogenously determined by technological parameters. The retirement of capital goods is an economic decision impacted by economic variables and market conditions. In this framework, capital retirement varies over time. Hulten [1990] pointed out that the assumption that retirements are independent of market conditions is one of the most serious problems in capital measurement. But traditional growth accounting studies estimate $TFP$ by assuming a constant capital lifetime. The resulting estimates of capital stocks and, finally, of $TFP$ growth, are thus biased. The issue at the stake here is to known whether such a bias can help to explain, at least partly, the puzzling $TFP$ dynamics. In order to shed some light on this issue, I develop a vintage capital model of economic growth built on Hendricks [2000] and Whelan [2002]. I extend their work in two complementary ways. First, the model developed is a two-sector model of economic growth that allows for different sectoral rate of technological change. This feature of the model is essential to grasp the causes and consequences of the unbalanced growth path observed in the US economy (cf. Whelan [2003]). Second, I numerically investigate the dynamic properties of the model in situations of unbalanced growth.

INVESTMENT-SPECIFIC TECHNOLOGICAL CHANGE AND THE RELATIVE PRICE OF CAPITAL

Interestingly, one can notice that the deceleration of productivity coincides in the US economy with a sudden break in the evolution of prices of equipment capital goods relative to consumption goods (see Figure 1). Following Whelan [2003], I interpret this persistent break as evidence that the two sectors do not share the same rate of technological progress. More precisely, since the mid-sixties, the rate of investment specific-technological change$^2$ seems to markedly exceed the rate of technological progress in the consumption good sector. Over

$^2$ Investment-specific technological change refers to productivity improvements in the production of new efficiency units of investment goods. This technological change can either be embodied in new vintages of capital or disembodied (cf. Ho and Stiroh [2001]).
the long run, however, nominal spending on investment and consumption have
tended to grow at the same rate. The declining relative price of investment
goods implies that real investment tends to grow faster than real consumption.
Clearly, these patterns are inconsistent with the traditional balanced growth
hypothesis.

This observation has already been formulated in a different framework by
Greenwood, Hercowitz and Krusell [1997] (GHK). These authors claimed that
embodied technical change could be identified by comparing the relative prices
of quality-adjusted efficiency units of investment and consumption goods.
Using a quality-adjusted producers' durable goods deflator based on Gordon
[1990] and the National Income and Product Account (NIPA) deflator for non
durable and services (excluding houses), GHK estimate that the relative price of
equipment declined at an average annual rate of 3.21 percent from 1954 to
1990. The main idea of these authors is to use the hedonic measures proposed
by Gordon to quantify the evolution of the quality of capital goods (i.e. to
measure embodied technological change) in order to compute a quality-adjusted
index of relative price of equipment.

**Figure 1:** Price of Nonresidential Equipment and Software relative to
Personal Consumption Expenditures (Index, 1996 = 1)

![Graph of Price of Nonresidential Equipment and Software relative to
Personal Consumption Expenditures (Index, 1996 = 1)](image)

Source: NIPA, Table 7.4 and Table 7.6, BEA.

The interesting point illustrated by the figure 1 is that even the official non
quality-adjusted price index published by the Bureau of Economic Analysis
(BEA) shows a marked decline in the relative price of equipment since the mid-
sixties. This result suggests that, during this period, investment specific
technological change allowed to reduce drastically the relative price of new
capital goods in two distinct ways. On one hand, the embodiment of new
technologies in successive vintages of equipment permitted to increase the
efficiency of capital without increasing its cost in the same proportion. On the
other hand, the higher rate of disembodied technological change faced in the investment sector relative to the consumption goods sector allowed to reduce the relative price of non quality-adjusted units of capital goods. As a result, real investment grew at a noticeably higher rate than real consumption. A direct consequence of this result is that the traditional one-sector model of economic growth provides a poor description of the long-run behavior of the US economy (cf. Ho and Stiroh [2001], Whelan [2003]). At least, a two-sector framework appears to be by far better suited for modeling this unbalanced growth path.

The main point I want to make clear in the following section is that calibrating multi-sector models on the bases of traditional capital goods price indexes can be very misleading. This statement came simply from the fact that the concept of capital stocks used by the BEA to construct aggregated price indexes differs from the one used in production and economic growth theory.

The relative prices of capital goods

The price index for nonresidential equipment and software published by the BEA is obtained by adding together prices of more than thirty detailed types of assets. This aggregation is done using the Fisher ideal index, which is the geometric mean of a Laspeyre price index and a Paasche price index

\[
F_t = \frac{\sum p_t q_{t-1} \times \sum p_{t-1} q_t}{\sum p_{t-1} q_{t-1} \times \sum p_t q_t}
\]

(1)

where the p's and q's represent prices and quantities of detailed components. The chain-type price index value for the period t is then expressed as:

\[
P_t = P_{t-1} F_t
\]

(2)

Dividing this index by the price index of personal consumption expenditures gives the relative price of equipment. In a competitive economy, this relative price can be interpreted as a measure of the efficiency with which which equipment goods can be produced. As equipment and software are capital goods, the evolution of this relative price is sometimes also interpreted as a measure of investment-specific technological change.

Notice that the price index of the equation (2) is based on the prices of detailed assets. Deflating nominal investment in equipment by this index gives the value of real gross investment in equipment. Net stocks can be estimated using the usual perpetual inventory method

\[
K_t = (1 - \delta_{e,t}) K_{t-1} + I_t
\]

(3)

where \(\delta_{e,t}\) and \(I_t\) denote respectively the average economic depreciation rate and the amount of real investment.
The aggregated net stock \( K_t \) is estimated by the BEA as a measure of real wealth stock. Alternatively, one can use the average physical depreciation rate instead of \( \delta_e \) in equation (3). This physical decay refers to the fact that a unit of capital becomes less capable of producing output as it ages. In this case, the aggregated capital stock obtained is often referred as *productive capital stock*. In general, physical decay and economic depreciation are different. A careful distinction is made between declines in the efficiency of an asset and the depreciation of that asset. However, if one supposes that capital physically decays at a constant geometric rate, it is well known that the rate of economic depreciation equals the rate of physical decay and, in this particular case, the distinction between wealth stock and productive stock simply evaporates\(^3\).

The issue at stake here is that these aggregated stocks are not well-suited measures of the capital input used in production. As it is well known, an important result of production theory is that it is desirable to aggregate capital goods in terms of their marginal products in current production as opposed to the marginal costs of producing the capital goods. In this framework, real capital stocks are estimated using the perpetual inventory method for detailed individual assets. The aggregated stock is then obtained by using implicit rental prices as weights. This approach, originated by Jorgenson and Griliches [1967], is based on the identification of implicit rental prices with marginal products of different types of capital. Estimates of these prices usually incorporate differences in assets prices, service lives, depreciation rates and the tax treatment of capital incomes. For an illustrative purpose, however, a simplified version of the implicit rental price formula for the asset \( i \) can be given by

\[
c_{i,t} = (r_t + \delta_{i,t} - \pi_{i,t}) p_{i,t}
\]

where \( r_t \), \( \delta_{i,t} \), \( \pi_{i,t} \) and \( p_t \) denote respectively the nominal rate of return on capital, the average rate of economic depreciation, the asset-specific capital gains term\(^4\) and the deflator for new capital goods.

The main effect of using rental prices as weights is to place relatively larger weights on assets which are depreciating quickly, compared to the weights that would result from a direct aggregation of stocks. The resulting aggregated capital stock is often referred as *capital services* by Jorgenson and its associates and as *capital input* by the Bureau of Labor Statistics (BLS)\(^5\).

Empirically, the capital input growth rate differs often substantially from the growth rate of the real capital stock. The relative price of equipment based on prices indexes published by the BEA is thus a very poor indicator of investment specific technological change. The proper indicator should measure the efficiency with which new units of capital input (not real capital stock) can be

\(^3\) On this point see, among others, Harper [1982] and Whelan [2002].

\(^4\) The asset-specific gains term is defined by \( \pi_{i,t} = (p_{i,t} - p_{i,t-1}) / p_{i,t} \).

\(^5\) See Dean and Harper [2001] for more details on the BLS productivity measurement program and methodology.
produced. Unfortunately, such an index is not yet available. The aim of the next section is to build such an indicator and to compare its evolution with the evolution of the relative price of equipment and software.

The relative price of capital input

There are two equivalent ways to build a price index of capital input. First, one can directly recalculate the Fisher ideal index of equation (1), taking the implicit rental prices to weight quantities and prices of detailed components. Deflating nominal investment by means of this index gives an index of the gross investment in capital input. Alternatively, one can begin by computing gross investment in capital input and then compute the price index as an implicit deflator. Gross investment in capital input can be obtained by aggregating real gross investment for individual assets in terms of their implicit rental prices. This aggregation can be performed using the Fisher formula or the Tornqvist index. Diewert [1992] examined these two index approaches to input measurement (as well as to output and productivity measurement). He concluded that there was an equally strong economic justification for Tornqvist and Fisher indexes but that the Fisher index was better suited when considering specific mathematical properties that have been suggested by various writers as desirable for an input quantity index. For this reason, I chose the Fisher formula to compute price and quantity indexes of capital input. All the data used in the derivation of real investment and implicit rental prices are taken from BEA and BLS estimates. More details on the methodology and source data can be found in the technical appendix.

Figure 2 depicts the resulting relatives prices for investment in capital stock and capital input. Of course, the relative price of capital stock is very similar to the relative price published by the BEA (cf. figure 1) because I used almost the same methodology and data to build it. The evolution of the relative price of capital input is, however, very different.

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6 A major advantage of the Fisher ideal index is that it has a "dual" property that is not shared by the Tornqvist formula. A Fisher ideal price index implies a Fisher ideal quantity index, and the converse. Thus, the product of a Fisher ideal price index between two periods and a Fisher ideal quantity index between the same two periods is equal to the total change in value between those periods. In contrast, a Tornqvist price index multiplied by a Tornqvist quantity index does not equal the change in value between the two periods (Tripelett [1992] p. 51).

7 Detailed estimates on real gross investment by industry and by type of assets are directly available on the BEA web site (http://www.bea.doc.gov). Data needed to build implicit rental prices can be downloaded from the BLS web site (http://www.bls.gov) at a more aggregated level of analysis. Disaggregated data used in this study have been kindly provided by the BLS upon request.
Figure 2: Relative prices for investment in real capital stock and capital input for Equipment and Software (Index, 1948 = 1)

Average annual growth rates of the relative prices can be found in Table 1. Relative prices increase at nearly the same rate until the sixties and then began to decline. This break in the trends of relative prices can be interpreted as a symptom of a shift in the sectoral composition of technological change.

Notice, moreover, that the decline in the relative price of capital input is by far faster than the decline of the relative price of capital stock. This is a direct consequence of the swift decline in the price of some equipment, which has led to a vast and continuing substitution of this equipment for other form of capital goods. As this equipment usually has a relatively short lifespan and a high depreciation cost, its implicit rental price is also relatively high. Hence, the index for the relative price of capital input gives a relatively higher weight to the components whose price falls quickly.

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8 See equation (4).
Table 1: Relative prices for investment in real capital stock and capital input for Equipment and Software (annual compound growth rates)

<table>
<thead>
<tr>
<th>Periods</th>
<th>Real Capital Stock</th>
<th>Capital Input</th>
<th>Non-IT Capital Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>1949 - 1960</td>
<td>1.55</td>
<td>-0.89</td>
<td>-2.32</td>
</tr>
<tr>
<td>1960 - 1970</td>
<td>-0.89</td>
<td>-1.83</td>
<td>-6.40</td>
</tr>
<tr>
<td>1970 - 1980</td>
<td>-0.87</td>
<td>-3.06</td>
<td>-6.17</td>
</tr>
<tr>
<td>1980 - 1990</td>
<td>-2.55</td>
<td>-6.17</td>
<td>-0.30</td>
</tr>
<tr>
<td>1990 - 1995</td>
<td>-2.55</td>
<td>-6.17</td>
<td>-0.55</td>
</tr>
<tr>
<td>1949 - 2000</td>
<td>-1.10</td>
<td>-3.50</td>
<td>-0.62</td>
</tr>
</tbody>
</table>

Source: Author's calculations based on BEA and BLS data sources

The last column of Table 1 shows the evolution of the relative price of capital input for equipment and software when IT related equipment is removed. When comparing the two last columns, it clearly appears that IT related equipment played a major role in the very quick decline in the relative price of capital input since the seventies. Notice however that the marked break in the trend of the relative price in the sixties is also observable in the non-IT capital input index.

UNBALANCED GROWTH AND PRODUCTIVITY SLOWDOWN

The sharp decline in the relative price of capital input in the mid-sixties coincides with a marked break in the pace of capital input accumulation. A plausible explanation for this coincidence is that technological advance has made equipment less expensive, triggering increase in the accumulation of this capital good. Figure 3 illustrates this sudden acceleration of investment.

Because of the impassioned current debate on the role played by information technologies (IT) in the recent resurgence of investment and productivity in the US economy, it is appealing to measure the contribution of IT in the sudden increase in investment in the sixties. For this purpose, growth rates of investment in capital input have been broken down into IT and non-IT components. Figure 4 clearly shows that IT played a negligible role in the change in the investment trend. This result seems to confirm that the pace of technological change accelerated markedly in the sixties, even in sectors producing non-IT equipment. The seventies, however, clearly mark the beginning of the huge process of input substitution driven by the sharp decline in the price of IT equipment. This input substitution, well documented by Stiroh [1998] and Jorgenson [2001] among others, is also noteworthy in Figure 4. Note that, thanks to the impressive acceleration in the accumulation of IT equipment,
the global investment in equipment *capital input* did not suffer so much during the mid-seventies economic crisis. It is also noticeable that, during this period, the average growth rate of investment in *non-IT* equipment *capital input* is still twice as large as what it was in the sixties.

**Figure 3:** Investment in real capital stock and *capital input* for Equipment and Software (Index, 1948 = 1, log scale)

![Figure 3](image1.png)

Source: Author's calculations based on BEA and BLS data sources

**Figure 4:** Contribution of *IT* and *non-IT* capital to investment growth in *capital input* for Equipment and Software (percent per year)

![Figure 4](image2.png)

Source: Author's calculations based on BEA and BLS data sources
The negative comovement between the relative price of capital input and the investment in capital input can be interpreted as evidence that there has been a significant acceleration in the pace of technological change in sectors producing capital goods. This result is however at odds with the collapse in TFP growth recorded by the voluminous literature devoted to the so-call productivity slowdown. For instance, according to Jorgenson and Stiroh [2000] and Jorgenson [2001], the annual TFP growth fell almost two-thirds of a percentage point from 1959-73 to 1973-90. Using Domar [1961]'s approach to decompose TFP growth by industry, these authors highlight the major role played by IT producing sectors in the global TFP growth. Their results, depicted by Figure 5, confirm the marked speeding-up of technological change in IT industries. But the most striking fact is undoubtedly the collapse of TFP growth observed in the rest of the economy for more than two decades. For the 1990-95 period, this TFP growth is even slightly negative.

**Figure 5:** Contribution of IT and non-IT industries to the US TFP growth (percent per year)

![Figure 5](image_url)

Source: Jorgenson [2001].

But, recently, the tables have turned. Much academic and popular opinion has moved toward the position that the benefits of the "new economy" revolution, based on computer hardware, software and other hi-tech products, are finally spilling over to the economy as a whole. One must recognize that the raw numbers are impressive. When the period since 1995 is compared to 1950-73 and 1973-1995, growth in output per hour in the third period has recovered more than two-thirds of the productivity growth slowdown registered between the first and the second periods. More detailed studies\(^9\) of this recovery clearly

\(^9\) See Gordon [2000], Oliner and Sichel [2000] and Jorgenson, Ho and Stiroh [2003].
show that almost all the acceleration of productivity gains observed are due to the use of IT and to a huge efficiency improvement in the computer-producing sector (including computer hardware, software and communication equipment). The rapid diffusion of IT appears to be a direct consequence of the swift decline in the price of computer-related equipment, which has led to a vast and continuing substitution of IT equipment for other forms of capital and labor. Real business investment in computer and peripheral equipment has jumped more than four-fold since 1995 (Oliner and Sichel [2000]). Consequently, the contribution of IT capital to output growth surged in the second half of the 1990s. The decrease of capital goods prices (especially for IT capital) is therefore at the heart of the capital deepening process that allowed the recovery of labor productivity. This is, moreover, a symptom of the obsolescence of old capital caused by the arrival of better, new capital goods (Greenwood and Jovanovic [2001]). These empirical evidences suggest focusing on the retirement of capital goods as an endogenous decision. As argued by Feldstein and Rothschild [1974], it seems to be really unrealistic to assume that retirements are determined exogenously when asset owners have the option of scrapping or otherwise disposing of assets in response to changes in relative prices or other market conditions (Cockburn and Murray [1992]). However, as stressed by Hulten [1990] and Berndt [1991], among others, given the importance of the capital depreciation process there is surprisingly little evidence concerning its actual character. Notable exceptions are the recent works of Hendricks [2000] and Whelan [2002]. In these two papers, the authors propose to incorporate explicitly the maintenance cost of capital in a vintage capital framework in order to model accurately the process of capital obsolescence. This framework appears to be a very promising way to endogenize depreciation patterns and service life of heterogeneous capital goods.

**A MODEL OF UNBALANCED GROWTH WITH CAPITAL GOODS OBSOLESCENCE**

This section aims at embedding the obsolescence mechanism in a two-sector model of economic growth. The first sector produces consumption goods and capital structures and benefits from disembodied technological change. The second sector produces equipment and software and benefits from both neutral and embodied technological progress. For simplicity, I focus on the private production side of the economy and ignore the government. Similarly, I do not dwell on the utility maximization problem of households.

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10 Oliner and Sichel [2000] estimate that the contribution of computer hardware to output growth during 1996-99 was about 0.6 % point per year, while the contribution of IT capital as a whole was 1.1 % points, a considerable step-up from the pace earlier in the decade.
The Production Technology

Following Solow [1960] and Phelps [1962], a constant return to scale production function is defined for each vintage of capital. I augmented the Solow’s original framework by allowing for disembodied technological change and multiple types of capital:

\[ Y_{C,v,t} = A_{C} L_{C,v,t}^\alpha J_{C,v,t}^{1/\alpha} S_{C,v,t}^{\rho_p} \]  
(5)

\[ Y_{K,v,t} = A_{K} L_{K,v,t}^\alpha J_{K,v,t}^{1/\alpha} S_{K,v,t}^{\rho_p} \]  
(6)

where \( \alpha + \rho_p = 1 \). \( Y_{X,v,t} \) is the output produced at time \( t \) using vintage \( v \) in the sector \( X \). \( L_{X,v,t} \), \( J_{X,v,t} \) and \( S_{X,v,t} \) denote respectively the amounts of labor, equipment and structures that work with vintage \( v \) of capital goods in the sector \( X \) (\( X = C, K, S \) respectively for consumption good, equipment and structures). Equipment capital stocks are measured in quality-adjusted units:

\[ J_{v,t} = q_J K_{v,t} \]  
(7)

The variable \( J \), which stands for jelly, refers to the assumption that capital goods of different vintages have to be perfect substitutes in production. The efficiency function \( (q_J) \) adjusts the quantity measure \( (K_{v,t}) \) for differences in marginal product due to embodied technological change and physical decay.\(^\text{11}\)

Firms allocate labor to each vintage \( v \) of equipment so as to maximize the profit generated by the vintage. According to first-order conditions, the optimal allocation of labor satisfies

\[ L_{C,v,t} = \left( \frac{P_{C,t}}{W_t} \right)^{1/\alpha} \left( \frac{\alpha_L}{\alpha_C} \right)^{1/\alpha} \left( \frac{\alpha_C}{\rho_p} \right)^{\rho_p} J_{C,v,t}^{\rho_p} \]  
(8)

\[ L_{K,v,t} = \left( \frac{P_{K,t}}{W_t} \right)^{1/\alpha} \left( \frac{\alpha_L}{\alpha_K} \right)^{1/\alpha} \left( \frac{\alpha_K}{\rho_p} \right)^{\rho_p} J_{K,v,t}^{\rho_p} \]  
(9)

where \( P_{X,t} \) is the price of one unit of good in sector \( X \) and \( r_{S,t} \) stands for the rental price of one unit of structures.

Capital Obsolescence

To capture the notion that the retirement of old capital goods is due to obsolescence, I follow Hendricks [2000] by assuming that retaining fraction

\(^{11}\) See Hall [1968].
\( \Delta_{v,t} \) of the equipment of vintage \( v \) requires flow of maintenance cost per surviving unit of \( m(\Delta_{v,t}) \). Retaining more equipment increases the maintenance cost \( m(\Delta_{v,t}) > 0 \). The purpose of this maintenance cost is to enable the model to accurately replicate the empirical age distribution of equipment vintages in use.

One interpretation of this specification is that each unit of capital consists of a continuum of individual "machines", each characterized by a time invariant idiosyncratic maintenance cost. Firms retire individual machines over time starting with those that cause the highest maintenance expenditures. Hence, the function \( m(\Delta_{v,t}) \) represents the expected maintenance cost conditional on survival. This maintenance cost is distributed according to a normal distribution truncated at zero, with mean \( \mu_m \) and standard deviation \( \sigma_m \). The optimal retirement age for each machine is then determined separately so that it maximizes the present value of revenue flows net of maintenance costs.

**Aggregation and Competitive Equilibrium**

Global values for inputs and output are obtained by aggregating quantities over vintages and sectors

\[
L_t = \sum_X L_{X,t} = \sum_{i=0}^V L_{c_{t-i},t} + \sum_{i=0}^V L_{k_{t-i},t} 
\]

\[
K_t = \sum_X K_{X,t} = \sum_{i=0}^V \Delta_{v_{t-i},t} L_{c_{t-i},t} + \sum_{i=0}^V \Delta_{v_{t-i},t} L_{k_{t-i},t} 
\]

\[
Y_{c,t} = \sum_{i=0}^V Y_{c_{t-i},t} \quad \text{and} \quad Y_{k,t} = \sum_{i=0}^V Y_{k_{t-i},t} 
\]

where \( I_{X,t} \) denotes real investment in the sector \( X \).

Market clearing condition implies

\[
Y_{n,t} = P_{c,t} Y_{c,t} + P_{k,t} Y_{k,t} + P_{s,t} Y_{s,t} + P_{m,t} M_t 
\]

where \( Y_{n,t} \) is the global nominal value of output and \( M_t \) the total real value of maintenance cost is defined by

\[
M_t = \sum_{i=0}^V \Delta_{v_{t-i},t} I_{c_{t-i},t} m(\Delta_{v_{t-i},t}) + \sum_{i=0}^V \Delta_{v_{t-i},t} I_{k_{t-i},t} m(\Delta_{v_{t-i},t}) 
\]

Finally, no-arbitrage in the capital rental market requires that the present value of the stream of rental payments for a capital good should equal the purchase price of the good. If \( r_t \) is the interest rate at period \( t \) and \( R_{v,t} \) stands
for the net cash flow obtained by using the vintage \( V \) at period \( t \), the no-arbitrage condition implies

\[
P_{r,t,t} = M a x \sum_{r=0}^{T} \left( \prod_{j=0}^{t} \frac{1}{1 + r_{j,t}} \right) R_{r,t,t}\tag{15}
\]

where \( T \) is the optimal service life of this machine.

The Steady State

The usual theoretical definition of a steady state is a situation in which all sectors grow at the same steady growth rate. In this model, this situation is highly unlikely. In general, the growth path is neither balanced nor steady. To illustrate this point, let us compute the growth rate of each sector

\[
\hat{Y}_{c,t} = \hat{A}_{c,t} \hat{L}_{c,t} \hat{q}^{a_{c}} \hat{K}_{c,t} \hat{S}_{c,t}\tag{16}
\]

\[
\hat{Y}_{k,t} = \hat{A}_{k,t} \hat{L}_{k,t} \hat{q}^{a_{k}} \hat{K}_{k,t} \hat{S}_{k,t}\tag{17}
\]

where \( \hat{X} = X_{t} / X_{t-1} \) is the growth factor of variable \( X \) at period \( t \).

Along a steady state growth path, investment and capital stock must grow at the same constant rate and the allocation of labor between the two sectors has to be invariable. Introducing such assumptions in equations (16) and (17) leads to:

\[
\hat{Y}_{c} = \hat{A}_{c} \hat{L} \hat{q}^{a_{c}} \hat{K} \hat{S}_{c}\tag{18}
\]

\[
\hat{Y}_{k} = \hat{A}_{k} \hat{L} \hat{q}^{a_{k}} \hat{K} \hat{S}_{k}\tag{19}
\]

It is easy to see from equations (18) and (19) that sectoral growth rates are generally different unless the two sectors share the same rate of neutral technological change (i.e. \( \hat{A}_{c} = \hat{A}_{k} = \hat{A} \)). In this very special case, however, the steady growth rate of each sector\(^{12}\) is given by:

\[
\hat{Y}_{c} = \hat{Y}_{k} = \hat{A} \hat{L} \hat{q}^{a_{c} + a_{k}} \hat{K} \hat{S}_{c}\tag{20}
\]

In this case, the two-sector model collapses to the usual one-sector growth model. This steady and balanced growth path will be considered as a benchmark when performing numerical experiments in order to study model dynamics.

\(^{12}\) Which is also the steady growth rate of the whole economy.
MODEL EXPERIMENTS

In this section, I examine the dynamics of the model by means of numerical experiments. The issue at the stake here is to analyze the impact of a changing sectoral composition of technological change on capital obsolescence and TFP measures. To clarify the main mechanisms at works in the model, I will first illustrate the effects of a technological shock that induces a permanent differential in the pace of technological progress between the two sectors. Then, in a second time, the model will be calibrated according to the evolution of the relative price of capital input presented earlier in this paper. The main issue will be to quantify the potential impact of the unbalanced growth path observed in the U.S. economy on the rate of retirement of equipment and, in turns, on the traditional measures of TFP.

Calibration

Before proceeding numerical experiments, few words are needed regarding the calibration of the model. At the beginning of each simulation, the economy is assumed to be on a balanced steady state growth path. The parameters’ values have been chosen in order to roughly fit post-war U.S. data. According to the steady state hypothesis, all the parameters aside from the ones determining the rate of technological change are keep constant over time. This allows to clearly disentangle the effects induced by the differential of technological pace from others induced by exogenous behaviors. In this perspective, parameters have been set to their average value on the period. More specifically, the shares in GDP of investment in equipment and of investment in structures have been rounded to, respectively, 10 and 5 percent of private nominal GDP. In the same way, the average growth rate of labor input is set to 1.5 percent per year. Other parameters concerning technology are presented in Table 3.

Table 3: Parameters

<table>
<thead>
<tr>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_L = 0.65$</td>
<td>Rounded average labor income share in GDP (BLS)</td>
</tr>
<tr>
<td>$\alpha_S = 0.13$</td>
<td>Taken from Greenwood, Hercowitz and Krussell [1997]</td>
</tr>
<tr>
<td>$\alpha_E = 0.22$</td>
<td>$\alpha_E = 1 - \alpha_L - \alpha_S$ (Constant return to scale hypothesis)</td>
</tr>
<tr>
<td>$\delta_t = 0.025$</td>
<td>Depreciation rate for structures (BLS)</td>
</tr>
<tr>
<td>$\tilde{q}_{1,t} = 0.95$</td>
<td>Exogenous physical decay for equipment</td>
</tr>
<tr>
<td>$\mu_m = 0.15$</td>
<td>Matches equipment age-survival profile and share of maintenance expenditures in GDP$^{13}$</td>
</tr>
<tr>
<td>$\sigma_v = 0.02$</td>
<td></td>
</tr>
</tbody>
</table>

$^{13}$ See McGrattan and Schmitz [1999] and Hendricks [2000].
The last three parameters of Table 3 concern the economic lifespan of equipment. The empirical literature typically assumes that retirements follow a modified Winfrey S3 distribution with an average service life of 12 years. Following Hendricks [2000], I chose the maintenance cost parameters so as to replicate this observed pattern (cf. Figure 6). I also adopted Hendricks' assumption according to which the maintenance costs are negligible during the first 5 years of equipment service life. I will soon go back on the meaning and consequences of this assumption.

**Figure 6:** Steady state age-survival profile for equipment

**Numerical results**

The aim of the following numerical experiments is to illustrate the impact of investment-specific technological change on the obsolescence rate of capital and TFP measures. Remember that, in this model, a balanced growth path exists only if disembodied technological change proceeds at the same rate in the two sectors (see equations (18) - (20)). In this case, the economy converges towards its steady state along which marginal productivity of new capital goods is constant over time\(^{14}\). In this case, the retirement rate of capital is also constant. On the contrary, if the two sector do not share the same disembodied technological rate, the unbalanced growth rate that prevails is characterized by a changing marginal productivity of capital goods that induces, at least temperately, a changing rate of capital retirement. To illustrate this statement, I

\(^{14}\) Notice that this convergence can be non-monotonic because of the so-call echo effect that characterizes vintage capital models (see Boucekkine, Germain and Licandro [1997]).
simulated four alternative scenarios of differentiated rate of sectoral technological change. To make comparisons easier, the overall TFP growth rate is set to 1 percent per year and is assumed to be constant over time. Results of these experiments are depicted by Figures 7 and 8.

In the first scenario, the rate of technological change in the consumption goods sector (which produces also structures) is supposed to be 1 % higher than the rate of disembodied technological change in the equipment-producing sector. In this case, the relative price of equipment \( \hat{P}_{i/c} \) increases at a constant rate of 1 percent by year. The consequences of this unbalanced path on the retirement rate of equipment and TFP measures are portrayed by Figures 7 (a) and 8 (a). The other figures show similar results for scenarios of decreases in the relative price of equipment goods of different magnitudes.

**Figure 7:** Retirement rate of equipment for alternative scenarios of evolution of the relative price of equipment \( \hat{P}_{i/c} \)

(a) \( \hat{P}_{i/c} = +1\% \)  
(b) \( \hat{P}_{i/c} = -1\% \)  
(c) \( \hat{P}_{i/c} = -2\% \)  
(d) \( \hat{P}_{i/c} = -3\% \)

These results can be interpreted in the following way. For a given constant share of investment in nominal GDP, a changing relative price of equipment modifies the rate of growth of real investment for this capital good. This change in the pace of capital accumulation impacts the marginal productivity of each vintage of equipment. If unit maintenance costs do not vary, the profitability of
each vintage will also change and so its retirement date. For instance, increases in the relative price of equipment slow down equipment accumulation. This relative scarcity of equipment increases its marginal productivity and, subsequently, delays its retirement. Arguments are symmetrical in the case of declining relative price of equipment. If TFP measurement assumes a constant retirement rate, estimates of levels and growth rates of equipment stocks are biased. The resulting TFP measures are then under or over-estimated, depending on the sign of the bias. Figure 8 (a)-(d) shows, for each scenario, the measured TFP growth rate (plain line) and the true growth rate (dotted line).

Figure 8: TFP mismeasurement for alternative scenarios of evolution of the relative price of equipment (\( \frac{I}{C} \))

Interestingly, Figures (c) and (d) suggest that the variation of retirement rate and so TFP mismeasurement can be temporarily. Over the long-run, retirement rate seems to stabilize around 18 percent per year. This result is directly linked with the assumption according to which maintenance costs are relatively low in the first years of equipment lifetime. In this case, the decrease in the marginal productivity of equipment accelerates the scrapping of old vintages because they are no more profitable. But this productivity decline has little impact on the retirement of recent vintages because of the low level of maintenance costs. The shortening of the average equipment lifespan reaches a threshold because of the non-continuity of the maintenance cost function. Of course, as long as one
considers that maintenance costs are low for recent vintages, but not nil, the
decline in marginal productivity ends to affect even young vintages and the
retirement rate of equipment increases again. This is exactly what Figures 7 (d)
and 8 (d) depict. The hypothesis of relatively low maintenance costs during the
first years of equipment lifetime is thus at the heart of the resurgence of
measured $TFP$. Notice that while the validity of such a hypothesis is difficult to
check rigorously at the macro level, it fits well with common observations of
everyday life. Everyone can remember that, in the past, the lifespan of durable
goods was longer. When these goods were breaking down, it was worthwhile
paying to fix the problem and continuing to use them for a while. Nowadays, no
one would accept to pay expensive maintenance costs to repair videotapes, TVs
or any other IT-related durable goods. Computers may be the most striking
example. Their prices are falling so fast that a break down is almost never fixed.
Computers are now discarded as soon as the first significant maintenance cost
appears.

Finally, a last numerical experiment is conducted by calibrating the model
with the relative price of equipment series observed in the U.S. The main
objective of this simulation is to check to what extent the mismeasurement of
$TFP$ induced by a shift in the sectoral composition of technological change can
account for the observed productivity puzzle. In this perspective, the overall real
productivity growth rate is arbitrary set to a constant 1 percent per year. Sectoral
composition of disembodied technological change is then determined according
to the evolution of the relative price of equipment presented in Table 1. The
results of this experiment are depicted by Figures 9 and 10.

**Figure 9: Retirement rate of equipment**

![Retirement rate of equipment graph]
At the beginning of the simulation, the economy is put on a balanced steady state in which the two sectors share the same rate of disembodied technological change (1% per year). Then, by period 50, the relative price of equipment is calibrated according to Table 1. During the first 10 years, the increase in this relative price depresses the accumulation of capital and finally induces an over-estimation of TFP growth. But in the sixties, the tables turned and the relative price of equipment began to decrease at an accelerated pace. The TFP growth is then markedly under-estimated. But the shortening of the lifespan of equipment slows down at the beginning of the nineties giving rise to a sudden resurgence of measured TFP growth.

**Figure 10: TFP mismeasurement**

In this very simple exercise, I do not claim to shed some light on the real value of TFP growth. It is worth remembering that the real TFP growth in the model is arbitrary set to 1 percent per year during the entire simulation time span. The similarity between Figure 10 and Figure 5 is however amazing. This result strongly suggests that the obsolescence mechanism put forwards by this simple model could play a major role in the explanation for the measured productivity slowdown and resurgence.

**CONCLUSION**

This paper suggests a two-sector model of unbalanced economic growth in which capital obsolescence is endogenous. In this framework, the fact that real investment grows at a higher rate than real GDP reduces the average service-life of equipment, which, in turns, induces a long-lasting underestimation of TFP.
growth by traditional growth accounting studies. Preliminary results obtained by
roughly calibrating the model to the U.S. postwar data show that this
mismeasurement could be substantial and may provide a good explanation for
well-known productivity puzzles.

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