Technical Change in Developing countries: A Dynamic Model of Adoption, Learning and Industry Evolution

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Abstract

This paper develops and analyses a dynamic model, which combines both the adoption and the industry evolution theories. We model the decision of adoption, learning entry and exit of firms. These decisions depend on the interaction of technology characteristics (effectiveness, machinery and information costs…) and other economic indicators (firm’s size, technology capability, competition concentration, returns of scale,…). We use the model’s theoretical results to analyze simultaneously the effects on the structure and the average efficiency of the industry and to develop a framework for understanding the public policy action necessary to enhance adoption and average productivity. The model we suggest also analyses effects on industry evolution and social welfare.

Keywords: Technological adoption, learning, efficiency, entry and exit, industrial dynamics, evolution, developing countries.

JEL- Classifications: L1, L11, L22, L25, O3, O31, and O33.

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1- INTRODUCTION

Much of the theoretical modeling in new growth theory has been in the context of the industrialized countries and focused primarily on R&D expenditures and investments in human physical capital as determinants of technological evolution. In developing economies, in contrast, where technologies are imported from industrialized countries, the R&D is oriented to the technological efforts that can enable firms to reach “best practice” levels of adopted technologies and that determine the intensity with which industrial technologies already used by firms are changed by continuing adaptation and incremental improvement.

Experience in developing countries indicates that these technical adoption and learning process are far more complex and demanding. The use of imported technologies, at or near “the best practice” level of technical efficiency for which it was designed, requires firms to seek new information, skills, material inputs, investment resources and management organizations. The adoption of innovations is not an automatic or passive process, in these countries, and the technological success of this adoption is uncertain.

Differences in firm-specific initial endowments of technological capabilities and entrepreneurial ability facilitate technological success by particular firms. Over time these firms learn more effectively than other enterprises and they may stay ahead or widen the technology gap. As a consequence, the technological adoption and learning processes, themselves, inevitably create technology gaps and affect the structure and the heterogeneity of the industry.

Developing countries appear to suffer from a significant technology gap between national and foreign firms. Part of this gap appears to be due to a great deal of heterogeneity in efficiency across firms in developing countries (especially in Africa). The main policies suggested are the intensification of the competition by exposing national firms to world competition and eliminating artificial restraints to competition such that barriers to entry; and the reallocation of resources away from less efficient firms to more efficient firms and sectors to improve aggregate productivity.

A key set of policy issues revolves around the relationship of inter-firm productivity differentials to firm size and employment creation. Researchers and policy makers have often associated the capacity to create employment with firm size. In developing economies, for example, microenterprises and small firms have often been viewed as important elements in the objectives of employment generation and poverty alleviation. To evaluate these policies and to find those which can have the greatest impact on increasing firm productivity and social welfare, we must focus simultaneously on both adoption and industrial evolution theories.

The purpose of this paper is to generalize previous studies by combining, both theories of adoption (Feder [1980], Fudenberg and Tirole [1985], Feder, Just and Silberman [1984], Jensen (1992), Hoppe (2000) and others), and industry evolution (Jovanovic [1982], Lippman and Rumelt [1982], Gort and Klepper [1982], Dixit and Shapiro [1986], Jovanovic and Lach [1989], Ericson and Pakes [1989],
1990), Lambson (1991, 1992), Hopenhayen (1992) D.B. Audretsch et Talat Mahmood (1994) and others). We present in the first section, a dynamic model of adoption and learning in which we formalize explicitly the firms’ entry and exit decisions, in a market for a differentiated product with monopolistic competition ¹.

These decisions depend on the interaction of the technology characteristics (innovation technical effectiveness, machinery and information acquisition costs, entry and adjustment costs,…) and the other economic indicators (firm’s size, technology capability, competition and concentration degrees, returns of scale,…) and affect the structure and the average efficiency of the industry.

In the second section we analyze theoretical results relative to firms’ adoption, efficiency and industry concentration and average productivity. The effects on industry evolution and social welfare are discussed in other paper.

2- THE MODEL :

2-1- Demand side:
I formalize a monopolistic competition model, using a derivation of Dixit and Stiglitz (1977) and Spence (1976). Each firm produces a unique brand of the same generic product. Hence, at any given time $t$, the number of firms operating, $n(t)$, equals the number of varieties available to consumers.

The preference ordering of identical consumers is described by the intertemporal utility function:

$$U = \int_{0}^{\infty} e^{-r t} (x_0(t) + \log C(t)) \, dt$$

Where $x_0(t)$ is the consumption of the numeraire in time $t$, and $C(t)$ is the consumption index of the Dixit-Stiglitz type

$$C(t) = \left( \int_{0}^{n(t)} y_j(t)^{\alpha} \, dj \right)^{1/\alpha} \quad \text{where} \quad 0 < \alpha < 1$$

Where $y_j(t)$ is the amount of variety $j$ of the differentiated product demanded by a consumer at time $t$. The aggregate demand function $Y_j(t)$ for variety $j$ at time $t$ is:

$$Y_j(t) = \frac{p_j(t)^{1/(\alpha-1)}}{n(t)} \int_{0}^{E} p_i(t)^{\alpha/(\alpha-1)} \, di$$

¹ The RPED World bank’s surveys found that the extent of imperfect competition exhibited in developing countries is relatively high. The majority of manufacturing sectors are operating under monopolistic condition, while the remaining are under oligopolistic condition.
Where \( E \) is equal to the total instantaneous expenditure on the differentiated product and \( p_j(t) \) is the price of variety \( j \) at time \( t \). The demand function (3) is isoelastic with the elasticity of demand \( \sigma = 1 / (1 - \alpha) \).

**2-2- Cost side:**

The technology used by the firm is described by the cost function:

\[
C^x_j(t) = c^x_j(t) y_j(t) + F.
\]

Where \( F \) is the fixed cost and \( c^x_j(t) \) is the marginal cost. Across firms \( c^x \)'s are random and take three possible values \( c^o_j(t) \), \( c^l_j(t) \) and \( c^h \) with \( c^h < c^l_j(t) < c^o_j(t) \). Firms experiencing \( c^o_j(t) \) are the lowest-efficiency (\( o \)-) firms, which still use the old technology. Those experiencing \( c^l_j(t) \), have adopted the new technology but they are still engaged in learning, adaptation and search efforts in order to succeed adoption and to use the new technology efficiently. Finally the high-efficiency (\( h \)-) firms which have achieved their successful adoption and learning process, use the new technology at the “best practice” level of technical efficiency for which it was designed (\( \hat{c}^h \)).

We assume that \( c^l_j(t) \) follows a conditional distribution \( F \left( c^l_j(t+1) / c^l_j(t) \right) \) which is the probability of having a productivity equals to \( c^l_j(t+1) \), in period \( t+1 \) given \( c^l_j(t) \) in period \( t \). \( F \) is continuous in \( c^l_j(t) \) and \( c^l_j(t+1) \), strictly increasing in \( c^l_j(t) \) and is the same for all firms. We define the probability of adoption success of firm \( j \), in period \( t \), by \( F \left( c^h / c^l_j(t) \right) \), which is the probability to use the new technology at the “best practice” level of technical efficiency (\( \hat{c}^h \)) in period \( t+1 \) given \( c^l_j(t) \) in period \( t \).

**Hypothesis 1:** We assume that \( c^o_j(t) = \hat{c}^o_j(t)^{-\theta} \) where \( c^o_j(t) \) can be considered as an indicator of the \( x \)-firm productivity, in period \( t \). Thus \( c^o_j(t) = \hat{c}^o_j(t)^{-\theta} \), \( c^l_j(t) = \hat{c}^l_j(t)^{-\theta} \) and \( c^h = \hat{c}^h^{-\theta} \).

**2-3- Market equilibrium:**

It is assumed that firms discover their type at the beginning of each period. A firm \( j \) of type \( x \) (\( x = o, l, h \)) which stays maximizes profits \( \pi^x_j(t) = p_j(t) y_j(t) - c^x_j(t) y_j(t) - F \), subject to the demand curve it faces given in (3). The optimal pricing rules for firm \( j \) of type \( x \) is:

\[
p_j(t) = \hat{c}^x_j(t) \left( \frac{\theta}{1 + \theta} \right) \text{where} \left( \frac{\theta}{1 + \theta} \right) \text{is the mark-up over the marginal cost and} \quad \theta = \frac{\alpha}{1 - \alpha}.
\]
Using this pricing rule, the profit expression of the firm \( j \) of type \( x \) is:

\[
\pi_j^x(t) = \frac{c_j^x(t) E}{(\theta + 1) cm_j n(t)} - F_t
\]

where \( cm_t = \frac{1}{n(t)} \int_0^{n(t)} c_j^x(t) \, dj \)

\( cm_t \) is the industry average productivity during this period. Thus \( cm_t^l \) and \( cm_t^o \), are respectively the average productivities of \((l)-\) and \((o)-\)firms in this period.

\[
\begin{align*}
cm_t^l &= \frac{1}{n^l(t)} \int_{n^h(t)}^{n^h(t) + n^l(t)} c_j^l(t) \, dj \\
\text{and} \quad cm_t^o &= \frac{1}{n^o(t)} \int_{n^h(t)}^{n^h(t) + n^o(t)} c_j^o(j) \, dj
\end{align*}
\]

**Hypothesis 2**: We assume that \( c_j^x(t) = A(j, t) cm \), with \( A(j, t) \geq 0 \) for all \( j \), is a continuous and monotonously decreasing function of the firm index \( j \). That is, firms are ranked in terms of this parameter in such a way that more efficient firms have a lower index number. We assume a specific functional form for \( A(j, t) \), namely:

\[
A(j, t) = 1 + \varepsilon(t) \left( \frac{1}{2} - \frac{j}{n(t)} \right) \quad 0 < \varepsilon(t) < 2
\]

Where \( \varepsilon(t) \) is an endogenous parameter measuring the industry concentration (or firms’ heterogeneity). We can see that higher values of this parameter imply a greater inter-firm variance in productivity and size. As \( \varepsilon(t) \) converges to zero the industry becomes homogenous and \( A(j, t) \) converges to 1.

Finally we can see that in the expression \( c_j^x(t) = A(j, t) cm \), the type of the firm does not matter. To make difference between \((l)-\) and \((o)-\)firms (which is necessary to avoid undetermined form and to solve the model) we assume that \( c_j^l(t) = l.A(j, t) cm \) and \( c_j^o(t) = o.A(j, t) cm \). Where \( l \) and \( o \) are two different positive values very close to 1 (logically \( l > o \)). This hypothesis does not affect results since \( l \) and \( o \) are instrumental variables which will disappear by simplification).

The expressions of the \((l)-\) and \((o)-\)firms profits can be written as follow:

\[
\pi_j^l(t) = \frac{l.A(j, t) cm_j E}{(\theta + 1)(c^h n^h(t) + cm_l n^l(t) + cm_o n^o(t))} - F_t
\]
\( \pi^o_j(t) = \frac{o \cdot A(j,t) \, cm^o_t \, E}{(\theta + 1) \left( c^h \, n^h(t) + cm^l_t \, n^l(t) + cm^o_t \, n^o(t) \right)} - F_t \) \hspace{1cm} (6)

2-4- The value functions

Potential entrants and incumbent firms maximize expected discounted profits. The problem of an incumbent firm using the old technology is defined recursively by:

\[
v^o_j(t, c^o_j(t)) = \pi^o_j(t) + \max \left\{ S^o ; \beta \left[ h_j(t) \cdot v^o_j(t+1, c^l_j(t+1)) + (1 - h_j(t)) \cdot v^o_j(t+1, c^o_j(t+1)) \right] \right\} \hspace{1cm} (7)
\]

Where \( v^o_j(t, c^o_j(t)) \) gives the value of a firm \( j \) of type \( o \), at period \( t \). \( S^o \geq 0 \) is the \( o \)-firm’s opportunity cost of being in the industry. \( h_j(t) \) is the hazard rate or the new technology adoption probability, of firm \( j \), in period \( t \). The value of firm \( j \) of type \( l \) in period \( t \), is:

\[
v^l_j(t, c^l_j(t)) = \pi^l_j(t) + \max \left\{ S^l ; \beta \int_{c^l_i(t)} c^l_j(t) \cdot v^l_j(t+1, c^l_j(t+1)) \cdot d c^l_j(t+1) \right\} \hspace{1cm} (8)
\]

Where \( S^l \) is the opportunity cost of being in the industry of \( l \)- and \( h \)-firms. \( S^l \) is assumed the same for an \( h \)-firm or an \( l \)-firm. \( F(c^l_j(t+1) / c^l_j(t)) \) is defined above. This \( l \)-firms become of type \( h \), in period \( t+1 \) if they succeed their adoption and learning process in period \( t \), i.e. \( F(c^h_j(c^l_j(t)) = 1 \). The \( h \)-firms’ value is:

\[
v^h_j(t, c^h) = \pi^h_j(t) + \max \left\{ S^h ; \beta \cdot v^h_j(t+1, c^h(t)) \right\} \hspace{1cm} (9)
\]

2-5- Industry dynamics:

The composition of firms evolves in accordance with average probabilities of adoption (\( o \)-firms), of technical success (\( l \)-firms) and of entry and exit (\( o \), \( l \) and \( h \)-firms). The number of \( h \)-firms evolves according to:

\[
n^h(t+1) = n^h(t) + p(t) \cdot n^l(t) - n^h(t) \hspace{1cm} (10)
\]

Where \( p(t) \) is the average probability of success of \( l \)-firms:

\[
p(t) = \frac{1}{n^l(t)} \int_{n^h(t)+n^l(t)}^{n^l(t)} F(c^h_j(c^l_j(t))) \, dj \hspace{1cm} (11)
\]
Thus $\rho(t)n^l(t)$ is the number of $l$-firms, which have achieved, with success, their adoption and learning process and become high-efficiency firms. $h(t)$ is the average probability of adoption of $o$-firms, in period $t$, then:

$$h(t) = \frac{1}{n^o(t)} \int_{n^h(t)+n^l(t)}^{n^o(t)} h_j(t) \, dj$$  \hspace{1cm} (12)

Let $n^o(t)$ the number of $o$-firms which adopt the new technology in period $t$, then

$$n^o(t) = h(t).n^o(t)$$  \hspace{1cm} (13)

The number of $o$-firms, $n^o(t)$, evolves according to:

$$n^o(t+1) = n^o(t) - n^a(t) + ne^o(t) - ns^o(t)$$  \hspace{1cm} (14)

$ns^o(t)$ is the number of exit among non innovating firms of type $o$, at the end of period $t$ (or at the beginning of period $t+1$). $ne^o(t)$ is the number of firms which enter the industry at the end of period $t$, using the old technology.

Let $ne^l(t)$ the number of innovating entrants of type $l$ and $ns^l(t)$ the number of exits among $l$-firms. The total number of $l$-firms in period $t+1$ is:

$$n^l(t+1) = (1 - \rho(t))n^l(t) + n^a(t) + ne^l(t) - ns^l(t)$$  \hspace{1cm} (15)

Finally, the total number of firms operating in the industry in period $t$, $n(t)$, is:

$$n(t) = n^h(t) + n^l(t) + n^o(t)$$  \hspace{1cm} (16)

This total number evolves in according to:

$$n(t+1) = n(t) + ne(t) - ns(t)$$  \hspace{1cm} (17)

Where $ns(t)$ is the total number of exits at the end of period $t$ (or at the beginning of period $t+1$):

$$ns(t) = ns^h(t) + ns^l(t) + ns^o(t)$$  \hspace{1cm} (18)

$ne(t)$ is the total number of entry at the end of period $t$ (or at the beginning of period $t+1$):

$$ne(t) = ne^l(t) + ne^o(t)$$  \hspace{1cm} (19)

### 2-6- The adoption decision:

A firm $j^a$ maximizes the discounted value of total profits by choosing the adoption date $T$.

Denoting the total profit function as $\Pi_{j^a}(T)$ the optimization problem of this firm is as follow:
\[ \Pi_{ja}(T) = \int_{0}^{T} e^{-rt} \pi_{ja}^{o}(t) \, dt + \int_{T}^{T+\tau} e^{-rt} \pi_{ja}^{l}(t) \, dt + \int_{T+\tau}^{+\infty} e^{-rt} \pi_{ja}^{h}(t) \, dt - e^{-rT} Xa(T) \]  

(20)

This profit function can be differentiated with respect to \( T \). One gets the first-order condition of the profit-maximization problem:

\[ \frac{d\Pi_{ja}(T)}{dT} = e^{-rT} (\pi_{ja}^{o}(T) - \pi_{ja}^{l}(T)) + e^{-r(T+\tau)} (\pi_{ja}^{l}(T+\tau) - \pi_{ja}^{h}(T+\tau)) - e^{-rT} (xa_{r} - rXa(T)) = 0 \]

(21)

Where \( Xa(T) \) is the adoption cost of the new technology in period \( T \). \( xa_{r} \) is the derivative of \( Xa(T) \) with respect to \( T \). \( T+\tau \) is the date of technical success of firm \( j^{a} \), such that \( F(c^{h}/c_{ja}^{l}(T+\tau-1)) = 1 \). At this date technical efficiency \( c_{ja}^{l}(T+\tau) \) equals \( c^{h} \), thus:

\[ \pi^{h}(T+\tau) = \pi_{ja}^{l}(T+\tau) \]

Eqs (21) writes:

\[ e^{-rT} \left[ \pi_{ja}^{o}(T) - \pi_{ja}^{l}(T) \right] = e^{-rT} (xa_{r} - rXa(T)) \]

(22)

If we replace in (22), \( \pi_{ja}^{o}(T) \) and \( \pi_{ja}^{l}(T) \) by their expressions (Eqs 6), we obtain (23).

\[ \frac{A(j^{a},T)(o-l)E}{(\theta+1)(c^{h}n^{h}(T) + cm^{l}_{r}n^{l}(T) + cm^{o}_{r}n^{o}(T))} = xa_{r} - rXa(T) \]

(23)

We can deduce from (23) that at any given date \( t \), there is a rank \( j^{a} \) such that condition (24) holds.

\[ (\theta+1)(c^{h}n^{h}(t) + cm^{l}_{r}n^{l}(t) + cm^{o}_{r}n^{o}(t)) = \frac{A(j^{a},t)(o-l)E}{(xa_{r} - rXa(t))} \]

(24)

We assume that \( j^{a} \) is the critical rank above which firm can not adopt the new technology, in period \( t \). Hence all \( o \)-firms which are larger and more efficient than firm \( j^{a} \) (i.e. which have lower rank than \( j^{a} \)), adopt in this period. The total number of firms which adopt in period \( t \) is given by Eqs (25):

\[ n^{a}(t) = j^{a} - (n^{h}(t) + n^{l}(t)) \]

(25)

One gets the expression of \( j^{a} \) given by (26).

\[ j^{a} = n^{a}(t) + n^{h}(t) + n^{l}(t) \]

(26)
2-7- The o-firms exit decision:
The exit decision is made prior to observing next period’s efficiency level and will involve a reservation rule:

\[ h_j(t) v_j^l(t+1, c_j^l(t+1)) + (1 - h_j(t)) v_j^o(t+1, c_j^l(t+1)) = S^o \]  

(27)

A firm using the old technology will exit the industry the first time its rank gets above this reservation value \( \hat{j} \), i.e. the first time \( j > \hat{j} \). Thus the number of o-firms exits, \( n_s^o(t) \), is given by:

\[ n_s^o(t) = n(t) - \hat{j} \]  

(28)

Finally there exists, for any given \( t \), a critical rank \( \hat{j} \) such that (29) is respected. (Eqs (29) is obtained by combining (6),(7) and (27)):

\[ (\theta+1)(c^h n^h(t) + cm^l n^l(t) + cm^o n^o(t)) = \frac{o.A(\hat{j}, t) cm_j E}{F_t + v_j^o(t, c_j^o(t)) - \beta} S^o \]  

(29)

2-8- The l and h-firms exit decisions:
The exit decision is made by h- and l-firms prior to observing next period’s efficiency level \( c_j^l(t) \) (or \( c^h \) if technical success) and will involve a reservation rule:

\[ \int_{c^h} v_j^l(t+1, c_j^l(t+1)) F(c_j^l(t+1) / c_j^l(t)) d c_j^l(t+1) = S^l \]  

(30)

A firm using the new technology will exit the industry the first time its rank gets above this reservation value \( j^* \), i.e. the first time \( j > j^* \). Thus the number of exits of l- and h-firms, \( n_s^l(t) \), is given by:

\[ n_s^l(t) = n^h(t) + n^l(t) - j^* \]  

(31)

We can see that if \( c_j^l(t+1) = c^h \) (i.e. \( j^* \leq nh(t) \)) the expected value of the firm \( j^* \),

\[ \int_{c^h} v_j^l(t+1, c_j^l(t+1)) F(c_j^l(t+1) / c_j^l(t)) d c_j^l(t+1) \]

becomes equal to the h-firm value, \( v_j^h(t+1, c^h) \). Hence this exit rule holds for the two types of (l-) and h- innovating firms. We can deduce too, that there is no positive h-firms exit when the number of l-firms surviving in the industry is positive, so \( n_s^l(t) = n_s^l(t) \) in this case.

Finally there exists, for any given \( t \), a critical rank \( j^* \) such that (32) holds. (Eqs (32) is obtained by combining (6),(8) and (30)):
(32)

\[
(\theta + 1)(c^h n^h (t) + c^m t^m n^m (t) + c^\omega n^\omega (t)) = \frac{1 \cdot A(j^*, t) cm_t E}{F_i + v^i_{j_i} (t, c^i_{j_i}) - \beta S^i}
\]

By combining (24) and (29) we obtain (33):

\[
\hat{j} = U^a (t) j^a - \xi (t) (U^o (t) - 1) n(t)
\]

Where, \( U^o (t) = \frac{(F_i + v^o_{j_i} (t, c^o_{j_i}) - \beta S^o) (o - l)}{o \cdot (x_{a_i} - r Xa (t))} \), and \( \xi (t) = \frac{2 + \epsilon (t)}{2 \cdot \epsilon (t)} \).

The number of exit of o-firms (Eqs 34) is obtained by replacing (33) and (26) in (28):

\[
ns^o (t) = \left[1 + \xi (t) (U^o (t) - 1)\right] n(t) - U^o (t) \left[n^a (t) + n^h (t) + n^l (t)\right]
\]

Substituting this in (34), the number of o-firms operating in the industry in period \( t+1 \) can be expressed as:

\[
n^o (t+1) = (n^h (t) + n^l (t)) \left[(U_o(t) - 1) (1 - \xi (t)) + e^o (t)\right] + n^a (t) \left[(U_o(t) - 1) (h(t) - \xi (t)) + e^o (t)\right]
\]

As in the earlier case, (36) is obtained by combining (24) and (32):

\[
j^* = U^i (t) j^a - \xi (t) (U^i (t) - 1) n(t)
\]

Where \( U^i (t) = \frac{(F_i + v^i_{j_i} (t, c^i_{j_i}) - \beta S^i) (o - l)}{l \cdot (x_{a_i} - r Xa (t))} \), and \( \xi (t) = \frac{2 + \epsilon (t)}{2 \cdot \epsilon (t)} \).

If we substitute (36) and (26) in (31), the number of innovating firms exit becomes:

\[
ns^i (t) = (n^h (t) + n^l (t)) (1 - U^i (t)) - U^i (t) n^a (t) + \xi (t) (U^i (t) - 1) n(t)
\]

The total number of l-firms becomes by replacing (37) in (15):

\[
n^l (t+1) = \left[(U^i (t) - 1) (1 - \xi (t)) + e^l (t)\right] n^h (t) + \left[U^i (t) (1 - \xi (t)) - \rho (t) + e^l (t) + \xi (t)\right] n^l (t)
\]

\[
+ \left[U^i (t) (h(t) - \xi (t)) + h(t) + e^l (t) + \xi (t)\right] n^o (t)
\]

2-9- Average probability of adoption \( h(t) \).

From Eqs(24) we can deduce:

\[
(c^h n^h (t+1) + c^m t_{t+1} n^m (t+1) + c^\omega n^\omega (t+1)) = \Omega^a (t)(c^h n^h (t) + c^m t^m n^m (t) + c^\omega n^\omega (t))
\]
Where \( \Omega^{a}(t) = \frac{k^{a}(j^{a}, t) \cdot (xa_{t} - r Xa(t)) cm_{t+1}}{(xa_{t+1} - r Xa(t+1))cm_{t}} \)

And \( k^{a}(j^{a}, t) = \frac{A(j^{a}, t+1)}{A(j^{a}, t)} \). \( k^{a}(j^{a}, t) \) is a positive endogenous parameter.

In the left side of (39) we replace \( n^{h}(t+1), n^{l}(t+1) \) and \( n^{o}(t+1) \) by their expressions given, respectively, by (10) (38) and (35). We obtain (40).

\[
n^{h}(t) \left[ c^{h} + cm_{t+1} \left[ (U^{i}(t) - 1)(1 - \xi(t)) + e^{l}(t) \right] + cm_{t+1}^{o} \left[ (U^{o}(t) - 1)(1 - \xi(t)) + e^{o}(t) \right] \right] + \rho(t) c^{h} + cm_{t+1}^{l} \left[ (U^{i}(t) - 1)(1 - \xi(t)) - \rho(t) + e^{l}(t) + \xi(t) \right] + cm_{t+1}^{o} \left[ (U^{i}(t) - 1)(1 - \xi(t)) + e^{o}(t) \right] = \Omega^{a}(t) \left[ c^{h} n^{h}(t) + cm_{t}^{l} n^{l}(t) + cm_{t}^{o} n^{o}(t) \right]
\]

Relation (48) holds for any positive \( n^{h}(t), n^{l}(t) \) and \( n^{o}(t) \), if system (41) is respected:

\[
\begin{align*}
\left[ c^{h} + cm_{t+1} \left[ (U^{i}(t) - 1)(1 - \xi(t)) + e^{l}(t) \right] + cm_{t+1}^{o} \left[ (U^{o}(t) - 1)(1 - \xi(t)) + e^{o}(t) \right] \right] & = \Omega^{a}(t) c^{h} \\
\rho(t) c^{h} + cm_{t+1}^{l} \left[ (U^{i}(t) - 1)(1 - \xi(t)) - \rho(t) + e^{l}(t) + \xi(t) \right] + cm_{t+1}^{o} \left[ (U^{o}(t) - 1)(1 - \xi(t)) + e^{o}(t) \right] & = \Omega^{a}(t) cm_{t}^{l} \\
+ cm_{t+1}^{l} \left[ U^{i}(t)(h(t) - \xi(t)) + h(t) + e^{l}(t) + \xi(t) \right] + cm_{t+1}^{o} \left[ (U^{o}(t) - 1)(h(t) - \xi(t)) + e^{o}(t) \right] & = \Omega^{a}(t) cm_{t}^{o}
\end{align*}
\]

The resolution of this system provides (see Appendix 1):

\[
k^{a}(j^{a}, t) = \frac{(1 - \rho(t)) \left( c^{h} - cm_{t+1}^{l} \right) (xa_{t} - r Xa(t+1))cm_{t}}{(c^{h} - cm_{t+1}^{l})(xa_{t} - r Xa(t+1))cm_{t+1}} \quad (41)
\]

\[
h(t) = \frac{\left( c^{h} - cm_{t+1}^{l} \right) \left[ c^{h} \rho(t)\xi(t) + x(1 - \rho(t)) cm_{t}^{o} \right] + c^{h} \xi(t)(cm_{t+1}^{l} - cm_{t}^{l}) + D}{(c^{h} - cm_{t+1}^{l}) \rho(t)c^{h} - c^{h} cm_{t}^{l} - cm_{t+1}^{l} \left( e c^{h} + 2 x cm_{t}^{l} \right) + D} \quad (42)
\]

Where \( D = (c^{h} - cm_{t+1}^{l}) \left( e^{l}(t) cm_{t+1}^{l} + e^{o}(t) cm_{t+1}^{o} \right) \) and \( x = \xi(t) - 1 \)

### 2-10 Innovating firms’ entry decisions:

An entrant \( j^{el} \), using the new technology, maximizes the discounted value of total profits by choosing the entry date \( T_{el} \). Denoting the profit function as \( \Pi_{j^{el}}(T_{el}) \), the optimization problem of this entrant is as follow:
The first-order condition of the profit-maximization problem is given by Eqs (44) below:

\[
\begin{align*}
\Pi_{j^{el}} (T_{el}) &= \int_{T_{el}}^{T_{el}+\tau} e^{-rt} \pi_{j^{el}}^{l} (t) \, dt + \int_{T_{el}+\tau}^{+\infty} e^{-rt} \pi^{h} (t) \, dt - e^{-rT_{el}} Xel(T_{el}) \\
\end{align*}
\]

The first-order condition of the profit-maximization problem is given by Eqs (44) below:

\[
\frac{d\Pi_{j^{el}} (T_{el})}{dT_{el}} = e^{-r(T_{el}+\tau)} \left[ \pi_{j^{el}}^{l} (T_{el}+\tau) - \pi^{h} (T_{el}+\tau) \right] - e^{-rT_{el}} \left[ \pi_{j^{el}}^{l} (T_{el}) + xel_{tel} - rXel(T_{el}) \right] = 0
\]

Where \( Xel(t) \) is the entry cost the firm must pay to enter using the old technology, in period \( t \). \( xel_{t} \) is the derivative of \( Xel(t) \) with respect to \( t \). \( T_{el}+\tau \) is the date of technical success of firm \( j^{el} \), such that \( F (c^{h} / c_{j^{el}}^{l} (T+\tau-1)) = 1 \). At this date, technical efficiency \( c_{j^{el}}^{l} (T+\tau) \) is equal to \( c^{h} \), thus: \( \pi_{j^{el}}^{l} (Tel+\tau) = \pi^{h} (Tel+\tau) \). The relation (44) becomes:

\[
\pi_{j^{el}}^{l} (T_{el}) = rXel (T_{el}) - xel_{Tel}
\]

If we replace \( \pi_{j^{el}}^{l} (T_{el}) \) by its expression we obtain:

\[
\Omega (\theta+1) \left( c^{h} n^{h} (T_{el}) + cm_{T_{el}}^{l} n^{l} (T_{el}) + cm_{T_{el}}^{a} n^{a} (T_{el}) \right) - F_{tel} = rXel (T_{el}) - xel_{Tel}
\]

We can deduce from (45) that, at any given date \( t \), there is a rank \( j^{el} \) such that condition (46)

\[
(\theta+1) \left( c^{h} n^{h} (t) + cm_{t+1}^{l} n^{l} (t) + cm_{t+1}^{a} n^{a} (t) \right) = \frac{\hat{c}_{j^{el}}^{l} (t) - \theta E}{F_{t} + rXel (t) - xel_{t}}
\]

We deduce

\[
(c^{h} n^{h} (t+1) + cm_{t+1}^{l} n^{l} (t+1) + cm_{t+1}^{a} n^{a} (t+1)) = \Omega (\theta+1) (c^{h} n^{h} (t) + cm_{t}^{l} n^{l} (t) + cm_{t}^{a} n^{a} (t))
\]

Where \( \Omega (\theta+1) = \frac{F_{t} + rXel (t) - xel_{t}}{F_{t+1} + rXel (t+1) - xel_{t+1}} \left( \hat{c}_{j^{el}}^{l} (t+1) \right) - \theta \)

In the left side of (47) we replace \( n^{h} (t+1), n^{l} (t+1) \) and \( n^{a} (t+1) \) by their expressions given, respectively, by (10), (38) and (35). We obtain the same system as (40), which provides the same
expression of adoption probability as (42) and the following expression for the endogenous parameter \( \Omega^{e^l}(t) \).

\[
\Omega^{e^l}(t) = \frac{(1 - \rho(t)) (c^h - cm_{t+1}^l)}{(c^h - cm_t^l)} \tag{48}
\]

2- The model’s results:

3-1- The average probability of technology adoption:

The simulation of the adoption probability expression (42) shows a positive impact of competition on the average probability of adoption see (figure 1-a). (This result confirms many others theoretic and empirical ones). The figure 1-b, however, describes a non-monotonic relation between the adoption of innovation and the industry concentration. It shows that a moderate concentration degree (lower than a critical value \( \epsilon(t) \geq 1.5 \) in figure 1-b)), raises the average probability of adoption which is consistent with the schumpeterian hypothesis according to which monopolistic profits are required to finance learning and R&D expenditures inherent to adoption process (Schumpeter [1942]). This effect becomes negative for a large variance in firms’ efficiency. Finally, in the figure 1-c we can see clearly the positive effect of the technical efficiency of the new technology, \( \hat{c}^h \), on the average probability of adoption \( h(t) \). Indeed, the more the innovation is drastic (\( \hat{c}^h \) is low), the more the adoption return is high.

3-2- The average productivity and probability of success:

The \( o \)-firms’ average productivity expression solved by the model (see appendix 2), is given by (49).

\[
cm_t^o \frac{2c^h \xi(t)}{1 + 3\xi(t)} \tag{49}
\]

The \( l \)-firms’ average productivity is:

\[
\frac{Ncm_t^l}{Dcm_t^l} \tag{50}
\]

Where:

\[
Ncm_t^l = 0.5 c^h \left[ e^l(t)(x + cm_{t+1}^o) \right] \left[ 2e^l(t)(f c^h \xi(t) + x v(x + cm_{t+1}^o)) - e^o(t) cm_{t+1}^o (ge^l(t) + h y) \right] \\
+ e^o(t)^2 \left[ 2yi e^l(t) - e x y^2 - d l e^l(t)^2 \right] + 2 c h e^o(t) cm_{t+1}^o e^l(t) \xi(t) (ke^l(t) - y j) + 4 c^h \xi(t)^2 e^l(t)^2 \\
+ 2 \sqrt{2} \left[ y \xi(t) e^l(t) e^o(t) cm_{t+1}^o (y + e^l(t)) \right] \left[ e^o(t) cm_{t+1}^o (x - e^l(t)) + e^l(t)(x + cm_{t+1}^o) \right]
\]
\[ q \left( (x + cm_{r+1}^o) (y e^o(t) cm_{r+1}^o + e e^h e^l(t)) + c h \xi(t) \left[ e^o(t) cm_{r+1}^o (y + 3 e^l(t)) - 2 e e^l(t) \right] \right) \]

And

\[ D c^m^l_i = 4 x e^l(t) (x + cm_{r+1}^o) \left[ e^o(t) cm_{r+1}^o (m y + d e^l(t)) + e^l(t) (2 c h \xi(t) + x (x + cm_{r+1}^o)) \right] + e^o(t)^2 cm_{r+1}^o (d e^l(t) - x y)^2 + 4 c h e^l(t) \xi(t) \left[ e^o(t) cm_{r+1}^o (n e^l(t) - 3 x y) + c h e^l(t) \xi(t) \right] \]

Where

\[ a = -16 \xi(t)^2 + 11 \xi(t) - 1, \quad b = 4 \xi(t)^2 - \xi(t) + 1, \quad c = 1 - 4 \xi(t), \quad d = 1 - 3 \xi(t), \]

\[ e = 1 - 2 \xi(t), \quad f = 6 \xi(t) - 5, \quad g = 24 \xi(t)^2 - 27 \xi(t) + 5, \quad h = 40 \xi(t)^2 - 41 \xi(t) + 5, \]

\[ i = 6 \xi(t)^2 - 6 \xi(t) + 1, \quad j = 9 \xi(t) - 8, \quad k = 8 - 3 \xi(t), \quad l = 6 \xi(t) - 1, \quad m = 1 - 5 \xi(t), \]

\[ n = 3 - \xi(t), \quad x = \xi(t) - 1, \quad y = \xi(t) + 1, \quad z = 2 \xi(t) - 3, \quad v = 4 \xi(t) - 3 \quad \text{and} \quad w = 2 \xi(t)^2 - 2 \xi(t) + 3 \]

The average probability of technical success of \( l \)-firms is:

\[ \rho(t) = \frac{A + cm_{r+1}^o \left[ cm^l_i (e^l(t) - e^o(t) - 2 x) - c h (e^o(t) + e^l(t) + 1) \right] + C}{B + cm_{r+1}^o \left[ c h (e - 2 e^o(t)) + C \right]} \quad (51) \]

where

\[ A = c h \left[ 2 cm^o_i (2 - \xi(t) + e^l(t)) + cm^l_i (z - 2 e^l(t)) \right], \]

\[ B = c h \left[ 2 cm^o_i (2 - \xi(t) + e^l(t)) + cm^l_i (z - 2 e^l(t)) \right], \quad C = 2 cm^o_i (x + e^o(t)) \]

The simulations of these expressions show (in figures 2-a and 2-b) that the probability of success and the average productivity of innovating firms are positively related (when firms’ productivity rises, the probability of success increases). They are both decreasing in the industry concentration degree over the interval \( 0, e(t)^* \) where \( e(t)^* \) is a critical value, above which the relation is positive.

Competition has two opposite effects on both average productivity and probability of success of \( l \)-firms: It has a positive effect since competition enhances innovating firms to invest in research and learning to succeed their adoption. And a negative effect since competition rises the probability of adoption which increases the number of less efficient firms of type \( l \) relative to the number of more efficient firms of the same type, which decreases the average productivity of \( l \)-firms. (We can see from figure 1-a, b and 2-a, b that the average probabilities of adoption and success are negatively related). The competition net impact is determined by the relative magnitudes of these two effects.
in the figure 2-b) when concentration is very high, and thus adoption probability is low, the positive effect dominate and competition increases the probability of success $\rho^3(t))$.

Finally, figures 2-c and 2-d show that the average productivity of $o$-firms is always lower than $l$ and $h$-firms’ ones and it is decreasing in the concentration degree (figure 2-d). However, a high competition enhances (or obliges) $o$-firms to adopt the new technology (see above) or to innovate marginally during the production process, while learning how to use more efficiently the current technology and human resources without adopting a radical innovation$^2$.

Finally, the innovation technical effectiveness is negatively connected to its probability of success (figure 2) because the learning process is longer and uncertain for complex technologies $(F(c^h/ c^l(t)))$ is decreasing in $c^h$.i.e. increasing in $c^h$.$^3$.

However this technical effectiveness of the new technology has a positive effect on the industry average productivity since it increases the average productivities of $l$- $h$ and $o$-firms (figure 2-f). This last result can be explained by knowledge spillover.

### 3-3- Industry concentration:

We can deduce from expression (41), which presents the general form of $k^a(j^a,t)$ for a firm $j^a$ the expression of $k^a(0,t)$ for firm 0 which is the first which adopt the new technology, i.e. when all firms operating in the industry use yet the old technology and the industry average productivity is thus equal to $cm^o_i$. One gets the expression of $k^a(0,t)$

$$k^a(0,t) = \frac{(1- \rho(t)) \left( c_h - cm^l_{t+1} \right) \left( xa_{t+1} - rXa(t+1) \right) cm^o_i}{(c_h - cm^l_j) \left( xa_t - rXa(t) \right) cm^o_{t+1}}$$  \hspace{1cm} (52)

To eliminate endogenous variables we begin by combining (47), (48) and (52) we obtain:

$$k^a(0,t) = \frac{(F_t + rXel(t) - xel_t) \left( xa_{t+1} - rXa(t+1) \right) cm^o_i}{(F_{t+1} + rXel(t+1) - xel_{t+1}) \left( xa_t - rXa(t) \right) cm^o_{t+1}} \left( \hat{c}_{jl}^l(t+1) \right)^{-\theta}$$  \hspace{1cm} (53)

---

2-Empirical analysis note that the great majority of the innovations was not necessarily the only result of adoption or R&D, but raise from marginal improvements of the equipment and the production organization

3-it has been found, for instance, that the best Korean firms have needed from 10 to 20 years to absorb complex capital goods technologies to the level of becoming internationally competitive
To eliminate $\frac{cm_i}{cm_{i+1}}$, we replace $cm_i$ by its expression (49), and by combining with (33) we obtain:

$$
\frac{cm_i}{cm_{i+1}} = \frac{(2+\varepsilon(t)) (5+\varepsilon(t+1))}{(2+\varepsilon(t+1)) (5+\varepsilon(t))}
$$

(54)

Finally if we replace $j_i^a$ by zero, in these two expressions: $A(j^a, t+1) = k^a_j(j^a, t) A(j^a, t)$ and

$$
A(j^a, t) = 1 + \varepsilon(t) \left( \frac{1}{2} - \frac{j^a}{n(t)} \right),
$$

we obtain:

$$
k^a_j(0, t) = \frac{(2+\varepsilon(t+1))}{(2+\varepsilon(t))}
$$

(55)

By combining (53), (54) and (55) one gets the recursive expression of the concentration degree

$$
\Xi(t+1) = \Psi(t) \Xi(t)
$$

where

$$
\Xi(t) = \frac{[2+\varepsilon(t)]^2}{(5\varepsilon(t)+6)}
$$

(56)

The general term of $\Psi(t)$ is given by:

$$
\Xi(t) = \Xi(0) \prod_{y=0}^{y=t-1} \Psi(y)
$$

where

$$
\Xi(0) = \frac{[2+\varepsilon(0)]^2}{(5\varepsilon(0)+6)}
$$

(57)

$\varepsilon(0)$ is the initial concentration degree at $t = 0$.

Finally from (56) and (57) we can deduce the (positive) expression of the industry concentration degree $\varepsilon(t)$:

$$
\varepsilon(t) = \frac{5 \Xi(t) - 4 + \sqrt{\Xi(t)(25 \Xi(t) - 16)}}{2}
$$

(58)

the industry concentration degree depends only on exogenous variables such as adoption and entry costs $Xa(t)$ and $Xel(t)$ (i.e. machinery, information, learning and adjustment costs), industry competition degree $\alpha$ and the efficiency level of innovating entrants, $c^{l}_{jel}(t)$.

We can see from (56) that an increase in the adoption cost in period $t$ relative to period $t+1$ involves an increase in $\Psi(t)$ and consequently rises the inter-firms variance in efficiency $\varepsilon(t+1)$, in period $t+1$. Indeed a raise in the adoption cost $Xa(t)$ in period $t$ relative to period $t+1$ dissuades incumbents and outsiders to adopt the new technology, at the end of period $t$, and thus to increase their efficiencies and market shares during the period $t+1$. It also enhances outsiders to enter with less expensive but
much less effective old technologies which involves a widening of the variation of the firms’ efficiency and size and thus an increase in the industry concentration degree during the period $t+1$.

If we consider that the adoption cost variation is unknown for the firms, their adoption behaviors will thus be characterized by an important economic and technological uncertainty, leading the firms to form anticipations. This anticipated variation of adoption cost $xa_t$ (which is the derivative of $Xa(t)$ with respect to $t$) has the opposite effect on the industry concentration degree. Indeed, in the case where $Xa(t)$ is decreasing in time, (i.e. $xa_t$ and $xa_{t+1}$ are negative) an important anticipated decrease in $Xa(t)$ during period $t+1$ (i.e. the absolute value of $xa_{t+1}$ increases), decreases the denominator of $\Psi(t)$ and consequently rises the industry concentration degree $\varepsilon(t+1)$, in period $t+1$. Since it enhances o-firms and innovating entrants to delay respectively their adoption and entry dates (to benefit from this cost decrease), and thus involves a large variance in firms’ efficiencies and sizes during period $t+1$ relative to $t$.

The net impact of adoption cost is determined by the relative magnitude of this two opposite effects. The figures 3-a, b and c describe, in a first case ($\varepsilon1$), the net effect of adoption cost, under perfect anticipation hypothesis (i.e. $xa_t$ is the real variation of adoption cost) and in the second case ($\varepsilon2$), under myopic anticipation hypothesis (i.e. $xa_t = 0$).

In the figure 3-a, where the adoption cost function is concave (i.e. $xa_t$ is increasing in $t$), firms anticipate at any date $t$ a more important decrease in adoption cost, during the period $t+1$, which enhances them to delay their adoption and entry decisions. This effect dominate during the first period when the value of adoption cost is very high (thus $\varepsilon1$ increases) and becomes zero when $Xa(t)$ falls sufficiently. However, in figure 3-b where $Xa(t)$ is convex (i.e. $xa_t$ is decreasing in $t$). The two effects are complementary. Finally in figure 3-c firms anticipate a durable increase in $Xa(t)$ which leads them to bring forward their adoption and entry dates. This positive effect can dominate the negative one if the adoption cost is very low, and becomes zero the first time it reaches a critical minimal value.

We find also that the adoption of innovations is more speed and the firms’ heterogeneity degree ($\varepsilon2$) is lower under myopic anticipation hypothesis, than under perfect anticipation one.

The entry cost $Xel(t)$ which includes the acquisition cost of the old technology and others administrative and adjustment costs, has exactly the same effects on firms’ heterogeneity as adoption cost $Xa(t)$. Important returns of scale (i.e. very high fixed cost), in period $t$, forms a barrier to entry and increases the industry concentration degree in period $t+1$.

Finally the more the innovating entrant $j^{el}$ is efficient at the end of period $t$ (i.e. $\hat{\varepsilon}^{j^{el}}(t)$ is low) the lower the variance in firms’ efficiency $\varepsilon(t+1)$, in the period $t+1$. 

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We can see that competition decreases the inter-firm variance in productivity at the period \( t+1 \) only if 
\[
\hat{c}_{j,l}^{\ell,t} (t) \leq \hat{c}_{j,l}^{\ell,t+1} (t+1)
\]
i.e. innovating at the end of period \( t \) are sufficiently efficient in the opposite case. 

Competition rises the industry concentration during the period \( t+1 \) (see Appendix 4 for demonstration). 

A more competitive environment may not be enough to sufficiently close the efficiency gap. Industry-specific training and technical assistance programs might help to reach this goal.

4- CONCLUSIONS

Our principal objective has been to explicitly model the decisions of adoption, learning, entry and exit of firms in a market for a differentiated product with monopolistic competition. We use the model's results to analyze simultaneously their effects on the average efficiency and the structure of industry in developing countries and to develop a framework for understanding the public policy action necessary to enhance adoption and average productivity. We have shown in this paper that industrial concentration affects the diffusion of new technology among firms. It has a positive effect on the average probability of adoption. (Which is consistent with the schumpeterian assumption, according to which monopolistic profits are required to finance research and learning expenditures); and becomes inhibiting when it reaches a high critical level. We can thus deduce that the significant inter-firm variation in technical efficiency in developing countries which is the source of average inefficiencies of their industries prevent the adoption of new technologies and thus the productivity improvement in this countries.

We found that an improvement of the productivity of less efficient entering firms towards that of domestic best practice by supporting their learning and research processes reduces the industry heterogeneity. Economies of scale (high fixed costs) rise industry concentration, which increases in the adoption, learning, and entry costs.

We have shown that competitive policy reform has a strictly positive effect on the average probability of adoption and lead to some firms moving toward best practice while overall inter-firm variance in productivity increases. Together with policy reform, industry-specific training and technical assistance programs might help to overcome this problem. Finally, we find that the innovation effectiveness increases the probability of adoption and decreases the average probability of technological success of innovating firms. Results concerning the dynamics of entry, exit and industry evolution are discussed in another paper.
Appendix 1

\[ c^h + cm^l_{r+1} \left[ (U^i(t) - 1)(1 - \xi(t)) + e^l(t) \right] + cm^o_{r+1} \left[ (U^o(t) - 1)(1 - \xi(t)) + e^o(t) \right] = \Omega^a(t) c^h \]  \hspace{1cm} (1)

\[ \rho(t)c^h + cm^l_{r+1} \left[ U^i(t)(1 - \xi(t)) + \rho(t) + e^l(t) + \xi(t) \right] + cm^o_{r+1} \left[ U^o(t)(1 - \xi(t)) + e^o(t) \right] = \Omega^a(t) cm^l_i \]  \hspace{1cm} (2)

\[ cm^l_{r+1} \left[ (h(t) - \xi(t)) + h(t) + e^l(t) + \xi(t) \right] + cm^o_{r+1} \left[ (U^o(t) - 1)(h(t) - \xi(t)) + e^o(t) \right] = \Omega^a(t) cm^o_i \]  \hspace{1cm} (3)

(1)-(2) implies \((1 - \rho(t)) (c^h - cm^l_{r+1}) = \Omega^a(t) ((c^h - cm^l_i) \text{ and } \Omega^a(t) = \frac{(1 - \rho(t)) (c^h - cm^l_{r+1})}{(c^h - cm^l_i)}\)

so \(k^a(j^a,t) = \frac{(1 - \rho(t)) (c^h - cm^l_{r+1}) (c^o(t) - c^l(t)) (\rho(t) - \xi(t))}{(c^h - cm^l_i) (c^o(t+1) - c^l(t+1)) (\rho(t) - \xi(t))}\)

Multiplying (2) by \((h(t) - \xi(t))\) and (3) by \((1 - \xi(t))\) gives respectively (2') and (3'):

\[ (h(t) - \xi(t)) \rho(t)c^h + cm^l_{r+1} \left[ (h(t) - \xi(t))(\rho(t) + \xi(t)) - (1 - \xi(t))(\rho(t) + \xi(t) + h(t)) \right] + e^o(t) cm^o_{r+1} (h(t) - 1)(h(t) - \xi(t)) + e^o(t) = \Omega^a(t) cm^l_i \]  \hspace{1cm} (2')

And

\[ (1 - \xi(t)) cm^l_{r+1} \left[ (U^i(t) - 1)(1 - \xi(t)) + e^l(t) + \xi(t) \right] + (1 - \xi(t)) cm^o_{r+1} (U^o(t) - 1)(h(t) - \xi(t)) + e^o(t) = \Omega^a(t) cm^o_i \]  \hspace{1cm} (3')

(2') – (3') gives (4'):

\[ h(t) = \frac{(1 - \rho(t)) (c^h - cm^l_{r+1}) \left[ \rho(t) c^h + \xi(t) + x(1 - \rho(t)) cm^o_{r+1} \right] + c^h \xi(t)(cm^l_{r+1} - cm^l_i) + D}{(c^h - cm^l_{r+1}) \rho(t) c^h - c^h cm^l_i - \rho(t) cm^l_{r+1}(c^h + 2 cm^l_i) + D}\]

Where \(D = (c^h - cm^l_i) (e^l(t) cm^l_{r+1} + e^o(t) cm^o_{r+1})\) and \(x = \xi(t) - 1\)

Appendix 2

There are 16 possible cases for entry and exit dynamic:

**Case 1:** is the general case presented in appendix 1: \(e^l(t) \geq 0, e^o(t) \geq 0, ns^l(t) \geq 0, ns^o(t) \geq 0\)

**Case 2:** Some \(l\) and \(o\)-firms enter but no exit: \((e^l(t) \geq 0, e^o(t) \geq 0, ns^l(t) = 0, ns^o(t) = 0\)

\[ c^h \left[ n^h(t) + \rho(t) n^l(t) \right] + cm^l_{r+1} \left[ (1 - \rho(t)) n^l(t) + e^l(t) n^o(t) + n^o(t) \right] + cm^o_{r+1} \left[ n^o(t) - n^o(t) + e^o(t) n(t) \right] = \Omega^a(t) \left[ c^h n^h(t) + cm^l_i n^l(t) + cm^o_i n^o(t) \right]\]

Which is equivalent to:

\[ n^h(t) c^h + cm^l_{r+1} e^l(t) + cm^o_{r+1} e^o(t) + n^o(t) \rho(t) c^h + \left(1 - \rho(t) + e^l(t) \right) cm^l_{r+1} + cm^o_{r+1} e^o(t) \]

\[ + n^o(t) \left[ cm^l_{r+1} (h(2(t)) + e^l(t)) + cm^o_{r+1} (1 - h(2(t)) + e^o(t)) \right] = \Omega^a(t) \left[ c^h n^h(t) + cm^l_i n^l(t) + cm^o_i n^o(t) \right]\]

This relation is respected for any \(n^h(t)\) and \(n^l(t), n^o(t)\) positive, in period \(t\), if and only if:
\[
\begin{align*}
\text{Case 6:} & \quad \Omega^a(t) = \Omega(t) + c^b \\
\text{Case 5:} & \quad \pm = \pm - c^b
\end{align*}
\]

Solving this system with respect to \( h_2(t) \) gives:
\[
\begin{align*}
& h_2(t) = \frac{(1 - \rho(t) + e^f(t) \pm + c^b \rho(t))^2}{(h_2(t) + e^f(t)) + c^o \pm (1 - h_2(t) + e^a(t))} = \Omega(t) \text{ \emph{com}_{j}}
\end{align*}
\]

As in the previous case, we solve this equation with respect to \( h_3(t) \):
\[
\begin{align*}
& h_3(t) = \frac{(h^m - c^m_l) (e^f(t) \pm + c^a(t) \pm + c^b \pm + c^o \pm + \rho(t))^2}{(h^m - c^m_l) \pm (h^m - c^m_l) + (h^m - c^m_l) \pm + c^o \pm (1 - \rho(t) + e^f(t)) + c^o \pm (h_2(t) + e^f(t)) + c^o \pm + c^b \pm (h^m - c^m_l) + c^o \pm (2c^m_l - c^b) - c^b \pm}
\end{align*}
\]

Case 4: Some l and o-firms enter but only l-firms exit:
\[
\begin{align*}
& h_4(t) = \frac{(h^m - c^m_l) \pm + c^o \pm + c^b \pm + c^o \pm + \rho(t))^2}{(h^m - c^m_l) \pm + c^o \pm (h^m - c^m_l) + c^o \pm \pm + c^b \pm (h_2(t) + e^f(t)) + c^o \pm (h_2(t) + e^f(t)) + c^o \pm + c^b \pm (h^m - c^m_l) + c^o \pm (2c^m_l - c^b) - c^b \pm}
\end{align*}
\]

Case 5: Some l and o-firms quit but only l-firms enter:
\[
\begin{align*}
& h_5(t) = \frac{(h^m - c^m_l) \pm + c^o \pm + c^b \pm + c^o \pm + \rho(t))^2}{(h^m - c^m_l) \pm + c^o \pm (h^m - c^m_l) + c^o \pm \pm + c^b \pm (h_2(t) + e^f(t)) + c^o \pm (h_2(t) + e^f(t)) + c^o \pm + c^b \pm (h^m - c^m_l) + c^o \pm (2c^m_l - c^b) - c^b \pm}
\end{align*}
\]

Case 6: No exit and only l-firms enter:
\[
\begin{align*}
& h_6(t) = \frac{(h^m - c^m_l) \pm + c^o \pm + c^b \pm + c^o \pm + \rho(t))^2}{(h^m - c^m_l) \pm + c^o \pm (h^m - c^m_l) + c^o \pm \pm + c^b \pm (h_2(t) + e^f(t)) + c^o \pm (h_2(t) + e^f(t)) + c^o \pm + c^b \pm (h^m - c^m_l) + c^o \pm (2c^m_l - c^b) - c^b \pm}
\end{align*}
\]

Case 7: Only l-firms enter and quit:
\[
\begin{align*}
& h_7(t) = \frac{(h^m - c^m_l) \pm + c^o \pm + c^b \pm + c^o \pm + \rho(t))^2}{(h^m - c^m_l) \pm + c^o \pm (h^m - c^m_l) + c^o \pm \pm + c^b \pm (h_2(t) + e^f(t)) + c^o \pm (h_2(t) + e^f(t)) + c^o \pm + c^b \pm (h^m - c^m_l) + c^o \pm (2c^m_l - c^b) - c^b \pm}
\end{align*}
\]

Case 8: Some l and o-firms enter but only l-firms quit:
\[
\begin{align*}
& h_8(t) = \frac{(h^m - c^m_l) \pm + c^o \pm + c^b \pm + c^o \pm + \rho(t))^2}{(h^m - c^m_l) \pm + c^o \pm (h^m - c^m_l) + c^o \pm \pm + c^b \pm (h_2(t) + e^f(t)) + c^o \pm (h_2(t) + e^f(t)) + c^o \pm + c^b \pm (h^m - c^m_l) + c^o \pm (2c^m_l - c^b) - c^b \pm}
\end{align*}
\]

Case 9: l and o-firms quit but only l-firms enter:
\[
\begin{align*}
& h_9(t) = \frac{(h^m - c^m_l) \pm + c^o \pm + c^b \pm + c^o \pm + \rho(t))^2}{(h^m - c^m_l) \pm + c^o \pm (h^m - c^m_l) + c^o \pm \pm + c^b \pm (h_2(t) + e^f(t)) + c^o \pm (h_2(t) + e^f(t)) + c^o \pm + c^b \pm (h^m - c^m_l) + c^o \pm (2c^m_l - c^b) - c^b \pm}
\end{align*}
\]

Case 10: No exit and only o-firms quit:
\[
\begin{align*}
& h_{10}(t) = \frac{(h^m - c^m_l) \pm + c^o \pm + c^b \pm + c^o \pm + \rho(t))^2}{(h^m - c^m_l) \pm + c^o \pm (h^m - c^m_l) + c^o \pm \pm + c^b \pm (h_2(t) + e^f(t)) + c^o \pm (h_2(t) + e^f(t)) + c^o \pm + c^b \pm (h^m - c^m_l) + c^o \pm (2c^m_l - c^b) - c^b \pm}
\end{align*}
\]
Case 11: Only o-firms enter and only l-firms quit: $e^l(t) = 0$, $e^o(t) \geq 0$, $n_s^l(t) \geq 0$, $n_s^o(t) = 0$

$$h_{11} = \frac{cm_{t+1}^o (c^h - cm_{t+1}^l) (1 + e^o(t) - \xi(t)) + (c^h - cm_{t+1}^l) p(t) \xi(t) c^h + cm_{t+1}^o (\xi(t) - 1) [1 - p(t)]} {(c^h - cm_{t+1}^l) (cm_{t+1}^l - cm_{t+1}^l) + \xi(t) c^h (cm_{t+1}^l - cm_{t+1}^l)}$$

Case 12: Only o-firms enter and quit, no entry or exit of l-firms: $e^l(t) = 0$, $e^o(t) \geq 0$, $n_s^l(t) = 0$, $n_s^o(t) \geq 0$

$$h_{12} = \frac{(cm_{t+1}^l - c^h) \left[ p(t) c^h \xi(t) - cm_{t+1}^o (1 - p(t)) (1 - \xi(t)) \right] + e^o(t) (cm_{t+1}^l - cm_{t+1}^l) + e^o(t) cm_{t+1}^o (c^h - cm_{t+1}^l)} {(c^h - cm_{t+1}^l) (cm_{t+1}^l - c^h) + \xi(t) c^h (cm_{t+1}^l - cm_{t+1}^l)}$$

Case 13: Some l- and o-firms quit but only o-firms enter $e^l(t) = 0$, $e^o(t) = 0$, $n_s^l(t) \geq 0$, $n_s^o(t) = 0$

$$h_{13} = \frac{\xi(t) (c^h - cm_{t+1}^l) (\xi(t) (1 - cm_{t+1}^l) + cm_{t+1}^l - p(t) c^h + 1)} {cm_{t+1}^o (c^h + cm_{t+1}^l) + (1 + p(t)) (1 - \xi(t)) (1 + cm_{t+1}^l)}$$

Case 14: No entry or exit: $e^l(t) = 0$, $e^o(t) = 0$, $n_s^l(t) = 0$, $n_s^o(t) = 0$

$$h_{14} = \frac{(c^h - cm_{t+1}^l) (cm_{t+1}^l - c^h) \rho(t) + (c^h - cm_{t+1}^l) (1 - \xi(t)) (1 + cm_{t+1}^l)} {cm_{t+1}^l - cm_{t+1}^l}$$

Case 15: No entry, only L-firms quit: $e^l(t) = 0$, $e^o(t) = 0$, $n_s^l(t) \geq 0$, $n_s^o(t) = 0$

$$h_{15} = \frac{cm_{t+1}^o (c^h - cm_{t+1}^l) (1 + \xi(t)) + (c^h - cm_{t+1}^l) p(t) c^h + cm_{t+1}^o (\xi(t) - 1) (1 - p(t))} {(c^h - cm_{t+1}^l) (cm_{t+1}^l - cm_{t+1}^l) + \xi(t) c^h (cm_{t+1}^l - cm_{t+1}^l)}$$

Case 16: No entry, only o-firms quit: $e^l(t) = 0$, $e^o(t) = 0$, $n_s^l(t) = 0$, $n_s^o(t) \geq 0$

$$h_{16} = \frac{(cm_{t+1}^l - c^h) \left[ p(t) c^h \xi(t) - cm_{t+1}^o (1 - p(t)) (1 - \xi(t)) \right] + \xi(t) c^h (cm_{t+1}^l - cm_{t+1}^l)} {cm_{t+1}^l - cm_{t+1}^l}$$

These particular cases imply that when $e^o(t) = 0$ we must have:

$h_1(t) = h_3(t)$, $h_2(t) = h_4(t)$, $h_4(t) = h_5(t)$, $h_5(t) = h_6(t)$, $h_6(t) = h_7(t)$, $h_7(t) = h_{13}(t)$, $h_{10}(t) = h_{14}(t)$, $h_{11}(t) = h_{15}(t)$, $h_{12}(t) = h_{16}(t)$

Among these eight conditions we find only five which always hold. These conditions are:

$h_2(t) = h_4(t)$, $h_3(t) = h_5(t)$, $h_4(t) = h_{14}(t)$, $h_{13}(t) = h_{15}(t)$, $h_{12}(t) = h_{16}(t)$

The other three conditions $h_{11}(t) = h_{12}(t)$ and $h_{13}(t) = h_{15}(t)$ are respected only if endogenous variables take some appropriate values.

We can also deduce that when $e^l(t) = 0$:

$h_1(t) = h_5(t)$, $h_2(t) = h_{10}(t)$, $h_3(t) = h_{11}(t)$, $h_4(t) = h_{12}(t)$, $h_5(t) = h_{13}(t)$, $h_6(t) = h_{14}(t)$, $h_7(t) = h_{15}(t)$, $h_{10}(t) = h_{16}(t)$

Among these eight conditions we find only five which always hold. These conditions are:

$h_2(t) = h_4(t)$, $h_3(t) = h_5(t)$, $h_4(t) = h_{14}(t)$, $h_{13}(t) = h_{15}(t)$, $h_{12}(t) = h_{16}(t)$

The other three conditions $h_{11}(t) = h_{12}(t)$ and $h_{13}(t) = h_{15}(t)$ are respected only if endogenous variables take some appropriate values.

Finally in the particular case where $e^l(t) = e^o(t) = 0$ we must have:

$h_1(t) = h_{16}(t)$, $h_{11}(t) = h_{16}(t)$, $h_{12}(t) = h_{15}(t)$, $h_{13}(t) = h_{14}(t)$, $h_{14}(t) = h_{12}(t)$, $h_{15}(t) = h_{13}(t)$, $h_{16}(t) = h_{10}(t)$

However we find that $h_1(t) = h_{16}(t)$ and $h_{13}(t) = h_{14}(t)$ do not hold for any endogenous variables values. Hence we have in all seven equations in height variables. The $8^{th}$ condition is deduced in appendix 3.

**Appendix 3:**

In this appendix we determine the total number of firms operating in the industry, $n(t)$.

We assume that $n(t+1) = g(t) n(t)$ where $g(t)$ is the growth rate of total number of firms, in period $t$.

So we have $n^h(t+1) + n^l(t+1) + n^o(t+1) = g(t) (n^h(t) + n^l(t) + n^o(t))$. By replacing $n^l(t+1)$, $n^h(t+1)$ and $n^o(t+1)$ by their expressions we obtain the following equation:

$$(n^h(t) + n^l(t)) \left[ U^o(t) + U^l(t) - 2(1-\xi(t)) + e^l(t) + 1 - g(t) \right]$$
This equation holds for any positive \( n^b(t) \) \( n^l(t) \) and \( n^o(t) \) if and only if:

\[
\begin{align*}
(U^o(t) + U^l(t) - 2)(1 - \xi(t)) + (e(t) + 1 - g(t)) &= 0 \\
(U^o(t) + U^l(t) - 1)(h(t) - \xi(t)) + (h(t) + e(t) + \xi(t) - g(t)) &= 0
\end{align*}
\]

(1)

(2)

(1)-(2) gives the expression of the growth rate of the total number of firms:

\[
g(t) = e(t) + \frac{h(t) + \xi(t)(1-2h(t))}{1-h(t)}
\]

thus \( n(t+1) = \left[ e(t) + \frac{h(t) + \xi(t)(1-2h(t))}{1-h(t)} \right] n(t) \)

We can see that if \( h(t)=0.5 \) then \( n(t+1) = (e(t)+1)n(t) = ne(t) + n(t) \). In this case the total number of exit is zero. We can deduce that in the cases 2, 6, 10 and 14 where \( ns^l(t) = ns^o(t) = 0 \). The probability of adoption must be equal to 0.5. One gets the 8th condition: \( h_z(t)=h_u(t)=h_{10}(t)=h_{14}(t)=0.5 \)

Appendix 4: as \( \theta = \frac{\alpha}{1-\alpha} \) is increasing in \( \alpha \), if \( \frac{\hat{c}_{jel}^l(t+1)}{\hat{c}_{jel}^l(t)} \geq 1 \), the term \( \frac{\hat{c}_{jel}^l(t+1)}{\hat{c}_{jel}^l(t)} \) is decreasing in \( \alpha \) and thus \( \varepsilon(t+1) \) falls with \( \alpha \). In the opposite case when \( \frac{\hat{c}_{jel}^l(t+1)}{\hat{c}_{jel}^l(t)} \leq 1 \), \( \varepsilon(t+1) \) rise with \( \alpha \).

REFERENCES

Figures

**Figure 1-a:** The competition effect on the average probability of adoption

- $h_1(t)$: low concentration
- $h_2(t)$: high concentration

**Figure 1-b:** The concentration effect on the average probability of adoption

- $h_1(t)$: low competition
- $h_2(t)$: high competition

**Figure 1-c:** Effect of innovation technical effectiveness on the average probability of adoption

- $h_1(t)$: low concentration and competition
- $h_2(t)$: high concentration and low competition
- $h_3(t)$: low concentration and high competition
- $h_4(t)$: high concentration and low competition

**Figure 2-a:** The concentration effect on the probability of technical success

- $\rho_1(t)$: low concentration
- $\rho_2(t)$: moderate concentration
- $\rho_3(t)$: high concentration

**Figure 2-b:** The effect of competition on the probability of technical success

- $\rho_1(t)$: low concentration
- $\rho_2(t)$: moderate concentration
- $\rho_3(t)$: high concentration
Figure 3: Evolution of industry concentration in adoption cost of innovation

Note: Xa (t) is the adoption cost of the new technology, ε1 is the concentration degree under perfect anticipation hypothesis and ε2 is the concentration degree under myopic anticipation hypothesis.