How to improve the efficiency of employment subsidies in a rigid labour market

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Abstract
This paper focuses on the economic complementarities between labour market institutions, namely firing costs, employment subsidies and a particular feature of European labour markets, the minimum wage. We investigate a matching model with endogenous job destruction in the tradition of Mortensen and Pissarides [1994]. Workers may be unemployed or employed in a stable or a subsidised job. Three aspects of the subsidy are studied: its duration, its amount and the proportion of jobs concerned. We show that its efficiency varies according to the way it is financed, the level of firing costs and is lower than without a minimum wage.

Keywords: Labour Market Policy, Unemployment, Matching model.
JEL-Codes: J38, J58, J68.
1 Introduction

During the last two decades, most industrialised countries have experienced a growing rate of unemployment, especially long-term unemployment. This high rate is often attributed to the institutional rigidities of the labour market. These rigidities prevent labour demand and labour supply from adjusting. Some economists advocate labour market reforms to cure this problem. However, “rigid” economies are characterised by politico-economic complementarities. According to Saint-Paul [1993,1997], labour market institutions like firing costs or minimum wage have created a status quo bias as their removal raises electoral costs\textsuperscript{1}. Active labour market policies such as hiring subsidies or decreases in taxes on low wages may then offer an option. If these policies are efficient in rising firms’ labour demand, they permit to postpone labour market reform as they offset the negative effects of institutional rigidities and specifically, firing costs. Firing costs deter firms from hiring as the latter consider it as an irreversible decision because of the costs they incur in the case of worker firing if the economic situation deteriorates (see Bertola [1990], Bentolila and Bertola [1990], Bentolila and Saint-Paul [1994] or more recently Mortensen and Pissarides [1999] in a matching framework). Most industrialized countries thus devote a sizeable part of their budget intended to cure unemployment to active labour market policies.

Two dimensions should be taken into account when conceiving such policies: their characteristics and the institutional setting in which they take place. Bélot and Van Ours [2000] point out that the consequences of labour market reforms depend upon the interactions with the other labour market institutions. The employment subsidies effects are then likely to vary according to the institutional setting. Using a matching model with endogenous job destruction similar to Mortensen and Pissarides [1994], this paper studies the comparative efficiency of a subsidy programme according to both its characteristics and the institutional setting in which it is implemented. Firstly, we show that the introduction of interactions between firing costs and minimum wage is likely to affect the consequences of employment subsidies. Secondly, we study the effects of several parameters of the subsidy policy like the proportion of job seekers placed in a programme or its duration.

Previous matching models simultaneously appraised the effect of firing costs and employment subsidies. However, they did not include a particular feature of many European labour markets: the minimum wage. These institutions seem to be complements as shown by some recent studies\textsuperscript{2}. In this paper, we have

\textsuperscript{1}Saint-Paul considers that the median voter is an unskilled worker. The latter supports firing costs even if they limit job creations as they allow him to keep his job and to extract a rent. The lower is his degree of unemployment exposure, meaning the more the labour market is rigid, the more he will favour it.

\textsuperscript{2}Bertola and Rogerson [1997] notice that job turnover rates between United States and European countries are quite similar in spite of institutional differences. They explain this convergence of results by the combination of high firing costs and wage levelling out measures for the latter. In fact, following an adverse shock on goods demand in the United States, firms
chosen to especially focus on France where a minimum wage and high firing costs coexist. Theoretical works devoted to employment subsidies using matching models usually conclude to their efficiency in dealing with unemployment (Mortensen [1996], Millard and Mortensen [1997] and Mortensen and Pissarides [2001]). Millard and Mortensen [1997], calibrating their model on the United Kingdom find out that a subsidy policy combined with a reduction in firing costs substantially decreases unemployment and raises the economic welfare. Mortensen [1996] simultaneously evaluates the effects of a hiring, a training and a wage subsidy. Hiring and training subsidies sufficiently decrease unemployment duration to offset the increase in its occurrence\(^3\), whereas wage subsidies reduce both unemployment duration and frequency at the same time. Finally, Mortensen and Pissarides [2001] focus on the consequences of these policies in a segmented labour market (by skills) and show that wage and employment subsidies increase employment and wages, particularly of less skilled workers. They take note of the fact that, as previously, hiring subsidies have an ambiguous effect on unemployment.

Our model also differs from models to date as we aim at studying the optimal design of employment subsidy as Orszag and Snower\(^4\) [2003]. However, the authors are only interested in the characteristics of the subsidy. They show that wage subsidies are more effective in reducing unemployment and lengthening job duration when workers have flat wage profiles whereas hiring subsidies obtain better results when workers experience wage increases. In our paper, we account for the complementarity between minimum wage, firing costs and employment subsidies. Previous matching models considered an increase in the amount of the subsidy and implicitly assumed that it was offered indefinitely to all workers. We drop this assumption and take the possible deadweight effect generated by the subsidy into account. This effect refers to a situation where firms decide to create the job only to benefit from the subsidy and end it when the latter stops. Recent empirical studies have shown that such an effect may be sizeable (see Meager and Evans [1998] and OECD [2001] for recent surveys). We thus define an extra condition, not identified by preceding matching models studying subsidies, of transformation of subsidised jobs into stable jobs (similar to the one defined by Cahuc and Malherbet [2003] or Cahuc and Postel Vinay [2002] in a different setting). Moreover, we examine several financing options for employment policy

\(^3\)Hiring subsidies lead to an upward shift of the job creation curve thus increasing labour market tightness (and reducing unemployment duration) and the reservation product of firms (and then, the number of destroyed jobs, measuring unemployment frequency).

\(^4\)Immediately adjust their labour force, firing costs being low. In Europe, the end of the wage distribution being truncated, firms, despite the high firing costs, decrease their labour demand as they are not able to give some of their employees the minimum wage. Job destruction is thus more important than if wages were in a position to adjust. Cahuc and Zylberberg [1999] study the comparative effects of firing costs under several minimum wage regimes. They find out that when a minimum wage prevails, the employment effects of firing costs are less favourable than those observed by Mortensen and Pissarides [1999], especially when the minimum wage is high and applies to all jobs because wages cannot decrease and limit the job regulations effects.
and show that the latter has an impact on its efficiency.

In the remaining of the paper, the analysis is structured as follows. In section 2, the model is described. Before studying labour market equilibrium, we determinate the job creation and destruction conditions. Sections 4 and 5 respectively display the qualitative and quantitative results of the model. As the qualitative effects of the policy are indeterminate, we are going to simulate the model for France to display the employment effects of subsidies. Section 6 concludes.

2 The model

The model developed here is similar to the matching model with endogenous job destructions of Mortensen and Pissarides [1994], [1999] extended by Cahuc and Postel-Vinay [2002] or Cahuc and Malherbet [2003] who have introduced an intermediate state5.

2.1 General labour market setting

In the general model of Mortensen and Pissarides, both job search by job seekers and job posting by firms have costs. Labour market information is incomplete and looking for an efficient match is costly for all the players involved. Jobs and workers’ matching is viewed as a production process. It takes the form of a matching function, jobs and workers being the inputs and the matches, the output. The matching function is assumed to present the usual properties. It exhibits constant returns to scale and can be written as $m(u, v)$ with $u$ being the rate of unemployment and $v$, the rate of vacancies (measured as shares of the labour force normalised to one). $m$ is increasing in $u$ and $v$, which implies that the number of matches increases when the number of vacancies or job seekers rises. In addition, $m(0, v) = m(u, 0) = 0$, there cannot be hiring if one of the argument is equal to zero (no vacancies or no job seekers in the labour market). Let $\theta = v/u$ be a measure of labour market tightness. The rate at which vacant jobs are filled is defined as:

$$h(\theta) = m(u, v)/v = m(\frac{u}{v}, 1) = m(\frac{1}{\theta}, 1)$$

with $\partial h/\partial \theta < 0$, which means that the rate at which a vacancy is filled is decreasing with labour market tightness. The rate at which job seekers exit from unemployment also depends on labour market tightness: $p(\theta) = m(u, v)/u = \theta h(\theta)$. The probability of getting a regular job depends on the probability for the firm of filling a vacancy. It is increasing with tightness indicator: $\partial p/\partial \theta > 0$.

The government chooses to implement a proportional subsidy policy and subsidises a share $\gamma$ of the new jobs created to give firms an incentive to hire

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5 The authors both differentiate stable employment from temporary employment.
A share $\mu$ of the subsidised jobs ends as the government decides during each time period. $R_0$ corresponds to the firm’s reservation productivity for subsidised jobs ending on a government decision or to the reservation productivity necessary for the firm to turn a subsidised job into a permanent one. It differs from $R_d$ because employers can then fire their employees without costs. Among the destroyed subsidised jobs, a share $\frac{1-\Phi(R_0)}{1-\Phi(R_s)}$ of the programme participants has a sufficient productivity to become regular workers and a share $\frac{\Phi(R_0)-\Phi(R_s)}{1-\Phi(R_s)}$ enters unemployment again (the denominator is due to the fact that only workers having a productivity higher than the reservation productivity for a subsidised job have remained employed following the preceding shock). Finally, a share $\lambda\Phi(R_s)$ of the jobs supposed to continue (still benefiting from government funding) ends because of an insufficient productivity of the recipients. In the steady state, all stocks have to be constant, which implies the following equilibrium constraints concerning unemployment $u$, regular employment, $n$ or subsidised employment,
\[ r: \]
\[
\mu \left[ \Phi(R_0) - \Phi(R_s) \right] + \lambda \Phi(R_s) \] \[ r + n \lambda \Phi(R_d) = u_p \tag{1} \]
\[
\mu \left[ 1 - \Phi(R_0) \right] r + (1 - \gamma) u_p = n \lambda \Phi(R_d) \tag{2} \]
\[
\gamma pu = \lambda \Phi(R_s) r + \mu r \tag{3} \]

### 2.2 Firms’ behaviour

We assume as Mortensen and Pissarides ([1994],[1999]) that jobs are hit by productivity shocks arriving at rate \( \lambda \). The match productivity, \( x \), which takes values on the unit interval, is distributed according to the distribution function \( \Phi(\cdot) \). Some job seekers have a too low productivity to be hired. We indeed assume that the wage paid by firms is exogenous which refers to a situation where the minimum wage applies to all jobs (Cahuc and Zylberberg [1999]).

The presence of a minimum wage may explain why firms are reluctant to hire low productivity job seekers as they are not allowed to decrease wages with worker’s productivity. This assumption of an insufficient workers’ productivity may be explained by the fact that this analysis applies to a particular segment of the labour market where workers are unskilled or have been unemployed for a long period\(^6\). The subsidy policy is thus a mean to compensate for the low productivity of these workers as it reduces their hiring and wage costs for firms. Moreover, subsidised workers are often paid the minimum wage.

The asset value of a stable job (indexed by \( d \)) for the firm having hired a worker with productivity \( x \), is defined as:

\[
\delta J_d(x) = x - \omega(1 + \tau) + \lambda \left[ \int_0^1 \max(J_d(\bar{x}), V - T)d\Phi(\bar{x}) - J_d(x) \right] \tag{4} 
\]

with \( \delta \), the discount rate. The value of having a filled job depends on the instantaneous gains, corresponding to the difference between the current worker’s productivity and the gross wage. Firms pay payroll taxes indexed on their wage bill, \( \tau \). The value of a filled job is also affected by the gains of the future period: the firm experiences a productivity shock \( \bar{x} \) with the arrival rate \( \lambda \) and then faces a trade-off. It may either fire the worker incurring firing costs or keep him earning \( J_d(\bar{x}) \). To take this decision, it accounts for the productivity value after the shock. Job security does not only affect the firm’s decision to maintain the

\(^6\)The workers’ productivity is indeed supposed to decrease with the duration of their unemployment spell, this phenomenon being attributed to the skill deterioration following a lasting period out of the labour market (workers are not in a position to upgrade their knowledge and lose their working habits).
worker on his job but also its hiring policy. The turnover costs will take two forms: a hiring cost, the firm having to look for a worker whose skills suit the job offered but also a firing cost when it decides to fire the latter. Let \( T \) be the firing cost paid by the firm.

If an employment subsidy is introduced, the firm has now the opportunity to hire workers whose productivity was too low to be hired at the prevailing wage in the presence of firing constraints. The subsidy \( \sigma \) is wage proportional and is paid during several periods. We assume that the subsidies are exclusively devoted to new firms to avoid an opportunist behaviour of the firms.\(^7\) In the case of a subsidy payment, the value of a filled job (indexed by \( S \)) becomes:

\[
J_S = \int_{R_0}^{1} J_{SH}(x)d\Phi(x) + \int_{R_0}^{R_0} J_{SB}(x)d\Phi(x) + \Phi(R_0)(V - \varsigma T) \tag{5}
\]

The subsidised job may take different forms: it may have a high productivity level sufficient to be turned into a stable job at the end of the subsidy period \( (J_{SH}) \), or its productivity may be too low and the subsidised worker must go back to unemployment \( (J_{SB}) \). The last job category refers to the subsidised jobs whose productivity is not high enough to be maintained after a productivity shock, the firm thus paying firing costs \( (\Phi(R_0)(V - \varsigma T)) \). The discounted value of a subsidised job with high productivity is defined as follows:

\[
\delta J_{SH}(x) = x - w(1 + \tau - \sigma) + \lambda (J_S - J_{SH}(x)) + \mu (J_d(x) - J_{SH}(x)) \tag{6}
\]

The subsidy adds to the firm’s gains of the current period in reducing the wage costs of its labour force. Following a productivity shock, the job may become less productive and even be destroyed if the productivity falls below a threshold under which the firm is not interested in continuing the employment relationship. If the firm separates from the worker although it is still entitled to the subsidy, it incurs firing costs \(^8\). These costs are assumed to be lower than those associated with a regular job and differ from the latter by a factor \( \varsigma \). The duration of the subsidy payment \( (1/\mu) \) depends on the government’s will. The latter decides when the benefitting firm will end receiving the subsidy. At the end of the subsidy period, the firm compares the gains generated by the pursuit of the job at the new productivity with those linked to the worker’s firing. The productivity threshold of the firm is then \( R_0 \). It is higher than \( R_d \) because the employer bears no cost in the case of separation. If the worker’s productivity

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\(^7\)In fact, the subsidy must not be paid to the firms having recently proceeded to firings. This could induce a replacing of the old workers by the workers benefiting from the subsidy, an effect identified as turnover effect by empirical studies. In the real world, it exists restrictions on the delivery of subsidies: firms cannot have fired workers in a recent past.

\(^8\)In France, when a firm hires a job seeker with a CIE (particular employment contract) and decides to break the contract for unjustified motives (in the absence of an act of negligence or in the case of serious difficulties), the subsidies received must be paid back.
is higher than this threshold productivity, he will remained employed, which is the case here because the subsidised job is endowed with a high productivity.

The subsidised job newly created may also have a low productivity and its asset value then takes the following form:

\[
\delta J_{SB}(x) = x - w(1 + \tau - \sigma) + \lambda [J_S - J_{SB}(x)] + \mu [V - J_{SB}(x)]
\]  

(7)

The essential difference with the preceding value function is that, at the end of the subsidy payment period, the employment relationship will terminate, the job having an insufficient productivity to be turned into a stable one. The subsidised worker will then become unemployed at rate \(\mu\). As previously, the productivity of the subsidised job may change following a productivity shock. The employment subsidy thus offers to the most disadvantaged job seekers an opportunity to restore their employability as their productivity may be increased during the payment of the subsidy.

Finally, the firm’s value of posting a vacancy is:

\[
\delta V = -k + (1 - \gamma) h(\theta)(J_d - V) + \gamma h(\theta)(J_S - V)
\]  

(8)

with \(k\) the per period cost of a vacant job borne by the firm. \(J_S\) is defined by equation (5) and \(J_d\) represents the gains distribution for a stable job, \(J_d = \int_{\tilde{R}_d} J_d(x) d\Phi(x) + (V - T) \Phi(\tilde{R}_d)\). During a time period, a firm sees a job seeker’s application for its job offer at the rate \((1 - \gamma) h\) and is proposed a subsidised worker by the Public Employment Service at the rate \(\gamma h\). This assumption may apply to the situation prevailing in France or in Belgium. Some firms first decide to hire a worker and in a second time ask for information to know if this hiring may benefit from a subsidy (see the empirical studies of Mahy, Nyberg and Mansson [1999] and Lizé [2000]). It also accounts for the fact that the government chooses to only subsidise a fraction of the new hires.

2.3 The budgetary constraint of the government

When the government implements its labour market policy, it must balance its receipts and outlays. We assume that outlays exclusively consist of active and passive spending on the labour market. Passive spending correspond to the benefits paid to the unemployed whereas active spending are defined as programmes to help the unemployed to get back to the labour market. Fiscal policy is restricted to the payroll taxes on wages paid by employers, firms being alone to bear the cost of the public employment policy. The budgetary constraint of the government is the following:

\[
(1 - u)\tau = u\rho + \gamma v h(\theta)\sigma
\]  

(9)
with $\rho$ being the replacement rate of unemployment benefit.

The latter thus faces a trade-off: it must allocate its receipts among job seekers’ compensation and the implementation of active programmes to improve the reemployment rate of the recipients. This budgetary constraint is simplified in comparison with the situation existing in France where active labour market policies and unemployment benefits are managed by different institutions\(^9\). We will study several scenarios of subsidy policy financing: either by an increase in the payroll taxes or a decrease in the replacement rate. This last possibility refers to the actual tendency of several European countries to reform their unemployment benefit system. The duration payment of unemployment benefits is limited and the payment of such benefits is replaced by a placement in an active programme when the unemployment spell has reached a given duration\(^10\).

We have not described the behaviour of workers as they are paid the minimum wage and they do not affect labour market equilibrium.

After having defined the behaviour of the various actors, we are interested in the equilibrium prevailing on the labour market when there is an employment subsidy and institutional rigidities such as firing costs and minimum wage.

### 3 Labour market equilibrium

The equilibrium prevailing on the labour market depends on one job creation condition and several job destruction conditions. We thus first identify the various job creation and job destruction conditions.

#### 3.1 Determination of the job destruction conditions

The various matches surpluses obtained by the firms are respectively for a stable job and a subsidised job:

\[
SF_d(x) = J_d(x) - V + T \\
SF_S(x) = J_S(x) - V + \zeta T
\]

According to the firms rationality assumption, the surplus of continuing the match must be higher than the separation cost. The various surpluses are monotonous increasing functions of the productivity, which means that job destruction satisfies the reservation property (Mortensen and Pissarides [1994]). So, there exists a single value of the reservation product which satisfies the job

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\(^9\) The insurance part of unemployment compensation is ruled by the UNEDIC, an organisation with equal representation of labour unions and firms. The unemployment benefit financing relies on payroll taxes paid by employers and employees. As far as active programmes are concerned, the Labour Ministry is involved and a share of the employment policy budget is affected to these policies.

\(^10\) This system was already existing in Sweden but Switzerland adopted it recently for the whole pool of its job seekers whereas the United Kingdom applied it only to the young unemployed persons in the New Deal’s setting and more recently, extended it to the long-term unemployed workers.
destruction condition, the latter implying a nil surplus for the firm. For firms having hired a worker with a long lasting contract, the reservation product of the match, $R_d$ satisfies the following condition:

$$J_d(R_d) - V + T = 0$$

According to the job creation condition, $V = 0$. Firms offer new vacancies until the discounted value of holding one equals zero, this result being linked to the profit maximisation behaviour of the firm. Thus, the job destruction condition amounts to $J_d(R_d) = -T$. Using the equation (4), accounting for the fact that $J_d'(x) = \frac{1}{x + \tau}$ and integrating by parts, we get the following stable jobs destruction condition:

$$R_d + \lambda \frac{\delta}{\sigma + \lambda} \int_{R_d}^{1} (x - R_d) d\Phi(x) = \overline{w}(1 + \tau) - \delta T \quad (10)$$

This relationship corresponds to the equality between the sum of the lower return on the match acceptable to the firm (the first term in the left side) and of the employer’s will to maintain the match in the case of a negative shock on productivity (if the firm experiences a negative shock, it can keep the job hoping a positive shock will occur during the next period) and the opportunity cost of the job. The first term in the right side refers to the job cost for the firm whereas the second one represents the cost of the contract breach (the firing costs paid). Thus, firing costs reduce the reservation product of firms because of the cost incurred by the latter when employers and workers separate. This equation allows us to define the productivity threshold of permanent jobs for firms.

The reservation productivity of the firm differs in the case of a subsidised job. We will thus establish several job destruction conditions according to the type of contract signed between the firm and the worker. As far as subsidised job is concerned, the employer and the employee decide to terminate the match only if the surplus value falls below those of firing costs, which leads to the following condition: $J_{SB}(R_s) = -\varsigma T$. The reservation threshold value is inferred from equation (7) and from the fact that $V = 0$:

$$R_s + \lambda \frac{\delta}{\sigma + \lambda} \int_{R_s}^{R_0} (x - R_s) d\Phi(x) + \lambda \frac{\delta}{\sigma + \lambda} \int_{R_0}^{1} (x - 1) d\Phi(x) - (1 + \tau - \sigma)\overline{w} + \delta + \lambda(1 - \Phi(R_0))\varsigma T + \lambda(1 - \Phi(R_0))J_{SH}(1) = 0 = \Gamma(R_s, \mu, \sigma) \quad (11)$$

Using equations (6) and (7) and the fact that $J_{SB}(R_s) = -\varsigma T$, the values of low and high productivity subsidised jobs may be written as follows:

$$J_{SH}(x) = \frac{x - 1}{\delta + \lambda} + J_{SH}(1) \quad \text{and} \quad J_{SB}(x) = \frac{x - R_s}{\delta + \lambda + \mu} - \varsigma T \quad (12)$$
Using these two relationships, we get the equation (11) defined above. The reservation productivity is higher for a stable job than for a subsidised job provided that the amount and the duration of the subsidy is sufficiently large (for the detailed proof see Appendix 1). The employment subsidy alters the job destruction condition in diminishing the value of the reservation productivity for the firms having hired unemployed workers in the case of a subsidy programme. The corresponding decrease in wage costs gives firms a further incentive to keep their workers. After the productivity shock, a subsidised job is preserved only if its productivity is higher or equal to $R_s$. The firing costs paid in the case of a contract breach during the period of subsidy receiving, also reduce this reservation productivity as does the fact that this job may become highly productive. $R_0$ equals the firm’s productivity when it decides whether or not to turn the subsidised job into a stable one following the subsidy period. Knowing that the firm will not incur firing costs if it fires the worker at the end of this last period, the firm faces the following destruction condition: $J_d(R_0) = 0$. Using the fact that $J_d(x) = \frac{x - R_s}{\lambda + \delta} - T$ (from equation (4)), we thus deduce this condition:

$$R_0 = R_\delta + (\delta + \lambda)T$$  \hspace{1cm} (13)

The reservation threshold of the firm at the end of the subsidy period is higher than the productivity required for the continuation of the employment relationship in the case of a permanent job. The firm anticipates that if it fires the worker following the end of the subsidy payment, it will not have to pay any compensation and the job value is thus reduced by this amount. The transformation condition for turning a subsidised job into a regular one is thus stricter than the conditions to fulfill to continue a permanent or a subsidised job as the job is no more protected by firing costs or a reduction in wage costs. We also have to define the value of the subsidised job with the highest productivity to solve the whole system, $J_{SH}(1)$. Using equations (6) and (12), we obtain:

$$(\delta + \mu + \lambda \Phi(R_0))J_{SH}(1) = 1 - \frac{1}{\lambda + \delta}(1 + \tau - \sigma) + \frac{\lambda}{\lambda + \delta + \mu} \int_{R_s}^{R_0} (x - R_s) d\Phi(x)$$  \hspace{1cm} (14)

$$+ \frac{\lambda}{\lambda + \delta} \int_{R_s}^{1} (x - 1) d\Phi(x) - T(\mu + \alpha \lambda \Phi(R_0)) + \mu \frac{(1 - R_\delta)}{\lambda + \delta}$$

### 3.2 Determination of the job creation condition

Firms supply vacant jobs until their expected value reaches zero ($V = 0$). From equations (6), (7), (8) and (12), we get the following job creation condition:
\[
\gamma \left[ \frac{1}{\lambda + \delta + \mu} \int_{R_s}^{R_0} (x - R_s) d\Phi(x) + \frac{1}{\lambda + \delta} \int_{R_o}^{1} (x - 1) d\Phi(x) - \varsigma T \Phi(R_0) + (1 - \Phi(R_0)) J_{SH}(1) \right] \\
+ (1 - \gamma) \left[ \frac{1}{\lambda + \delta} \int_{R_d}^{1} (x - R_d) d\Phi(x) - T \right] - \frac{k}{h(\theta)} = 0 = \Psi(\theta, \mu, \sigma, \gamma)
\]

This equation states a decreasing relationship between labour market tightness and the reservation productivity of firms having hired an unemployed worker under a regular contract \( R_d \) (using equations (10), (11), (13) and (14) (for the demonstration see the Appendix 1)). When the reservation productivity for stable jobs increases, job destructions are more numerous and job creations decrease. Firms anticipate that the new jobs will have a shorter duration which limits job creations. Thus, when firms are more demanding concerning workers’ productivity, labour market tightness falls. This curve corresponds to the job creation curve of Mortensen and Pissarides’ [1994] model. The government uses the programme participation rate to favour job creation by firms. The proportional subsidy directly intervenes in the job creation condition in the term \( J_{SH}(1) \) and increases the value of the subsidised job.

Finally, the model’s equilibrium is obtained the following way: equations (10), (11) and (13) allow us to deduce the reservation productivities of the stable and subsidised jobs \( R^*_s, R^*_0 \) and \( R^*_d \). Then, the job creation condition, the equilibrium conditions between the entry and exit flows of unemployment, subsidy programme and stable jobs (1), (2) and (3) determine the equilibrium vacancy rate \( v^* \), the equilibrium unemployment rate \( u^* \), the stable employment rate \( n^* \) and the subsidised employment rate \( r^* \) of this segment of the labour market. In the next section, we will assess the theoretical effects of the various parameters of employment policy.

4 The qualitative impact of the implemented policy

The government may use the different parameters of the employment policy to rise the employment level in the economy. It may affect the proportion of subsidised jobs, increase the amount of the subsidy or lengthen the subsidy payment period. We are first going to investigate the effect of these parameters on the job creation and job destruction curves before studying their global effects on the budgetary constraint, unemployment and regular employment. The various impact of these parameters on labour market tightness and on the various reservation productivities are summed up in the following table (\( \tau \) and \( \rho \) refer to the financing option chosen by the government):
The government may modify the job seekers participation rate and thus alter the distribution of the new jobs created between subsidised jobs and regular jobs. The equilibrium of the labour market is affected the following way: increasing the number of job seekers included in a programme leads firms to create more vacancies as subsidised jobs give firms a greater surplus than non subsidised jobs (see equation (15)). This is not only due to the amount of the subsidy but also to the fact that the costs borne in the case of a separation are less important (subsidised jobs are less protected than permanent jobs). The job creation curve shifts upwards inducing an increase in labour market tightness (for the proof see Appendix 2). The proportion of subsidised jobs has no direct effect on the job destruction conditions but an indirect effect through the budgetary constraint in the case of a tax financing. As the government places more job seekers into subsidy programmes, it increases its spending and raises the tax rate incurred by firms. This brings about a shift upward of all the jobs destruction curves (for the demonstration see Appendix 2). Following the rise of the wage burden, the job surplus becomes less sizeable and it is thus less profitable for firms to preserve their employment relationship. The net effect on reservation thresholds is unambiguously positive whereas the effect on tightness is indeterminate. Nevertheless, at this stage, we do not include the positive effects of subsidised job creation on global employment and the reduction in the costs of unemployment benefits generated by the employment subsidies if they are efficient. Moreover, the second effect is not present if the programmes are not financed by taxes but by a reduction in the replacement rate.

The policy maker may also affect the amount of the subsidy received by firms to induce them to hire more disadvantaged unemployed workers. If the amount of the subsidy is increased, the job destruction condition for subsidised jobs is modified. The reservation product necessary for the match to continue decreases and the job destruction curve for these jobs shifts downwards (for the proof see Appendix 2). Thus, labour market tightness gets higher. Nevertheless, in the case of a tax financing, this shift entails a rise of payroll taxes which increases all reservation productivities. The move of the subsidised jobs destruction curve is then indeterminate. The subsidy acts indirectly upon the job creation curve through the value of a high productivity job \( J_{SH}(1) \) and moves it upwards. A rise of the subsidy thus translates into a rise in \( R_0 \) and \( R_d \) in the case of a tax financing and an indeterminate effect on tightness and \( R_s \). In the case of a replacement rate financing, the two first productivities are left unchanged and \( R_s \) falls.
Finally, the government may decide to lengthen the duration of the spell during which firms receive the subsidy. The subsidised job must last sufficiently to allow the job seeker to learn the skills characterising his job but must not exceed a given duration beyond which he is locked in his programme and does not look for a job on the regular jobs market. Moreover, in the long run, the firm is likely to integrate the policy into its hiring behaviour, substituting a subsidised worker to another (the so-called "windfall effect"). A decrease in the rate at which subsidised jobs come to an end brings about a shift downward of the job destruction curve defined by equation (11) thus inducing a fall in the reservation product $R_s$. It also entails a move upwards of the job creations curve (see Appendix 2 for all details about the derivatives). When the government decides to reduce the termination rate of subsidised jobs, it raises the expected profits of such jobs and leads firms to increase the number of their vacancies. The net result of these shifts is a rise of labour market tightness and a reduction in the subsidised job productivity. Moreover, the increase in the duration of the subsidy payment infers a rise in the payroll taxes in the case of a tax financing and so the various labour market reservation productivities get higher.

As in most enunciated cases, we cannot conclude on the impacts of policy parameters on the various labour market equilibrium values, we resort to a simulation to see the magnitude of these different effects. Moreover, we will assess the efficiency of employment subsidies according to the level of job security because as mentionned in the introduction, these policies are complementary.

5 The quantitative impact of the policy

The matching function takes the form of a Cobb-Douglas function, $m = u^v v^{1-v}$, with $\nu = 0.5$ following Mortensen and Pissarides [2001]. The time unit is the quarter. The model is calibrated for an unemployment rate of 15%, this rate representing the situation of the less skilled workers in the French labour market (INSEE [2003]). The discount rate $\delta$ is fixed at 0.0125. The unemployment benefits are supposed to be wage proportional with a replacement rate, $\rho = 0.76$, which is the rate prevailing in France. We consider two alternative cases for the firing costs of stable jobs: high firing costs $T_H = 2w$ and low firing costs $T_L = 0.5w$. We have calibrated the model to find out the value of the minimum wage in the economy $w$, the cost of posting a vacancy $k$, the arrival rate of shocks $\lambda$ and the taxation rate $\tau$ (taking into account the fact that mean duration of an unemployment spell in France is 12.8 months (see INSEE [2002])). $k$ takes the value 0.5 which is in the range of the values existing in the literature. The cost of breaking the contract for the employers employing a subsidised worker is assumed to amount the level of the subsidy as we are not able to evaluate precisely the value of firing costs (we do not know when the contract breach occurs). The values of the various policy parameters in the benchmark situation are the following: $\gamma = 0, 1$, $\sigma = 0, 1$ and $\mu = 0, 5$. In the next subsections, we will
focus on the relative efficiency of employment subsidies according to the level of firing costs, high or low and to the different financing options for employment policy.

5.1 The case of a tax financing

We first study the effects of the various parameters of an employment subsidy policy with a tax financing. We assume that there is a binding budgetary constraint and that the tax increase cannot exceed the initial tax level by more than 10%. We show that this policy does not decrease unemployment except when the government decides to raise the amount of the subsidy. Our results thus differ from those obtained by Mortensen [1996] and Mortensen and Pissarides [2001].

In the case of a tax rate financing, a rise of the number of subsidised jobs is not efficient. The magnitude of the employment effects varies with the level of labour market flexibility. The regular employment rate falls by 2 percentage points and 2.5 percentage points respectively for high and low firing costs (see figure 2) and unemployment rises by 1.2, 2 percentage points in both cases. Subsidised employment increases by more in a flexible labour market because the subsidy policy entails more job creations (as the policy effect on the job creation curve decreases with the level of firing costs (see Appendix 2)). However, a rise of job destructions due to the increase in the various reservation productivities occurs and the additional job creations brought about by the implementation of the subsidy are not enough to compensate for them. This result is due to the existence of a minimum wage as less-skilled workers wages are not in a position to adjust to account for the rise in the tax rate. The rise in the tax rate thus allows the government to finance an increase in the subsidised jobs rate of 0.22 in the case of low firing costs instead of 0.18 when firing costs are high.

When the amount of the subsidy is raised, the policy consequences are more different between flexible and rigid labour markets. If firing costs are high, an increase in the share of the wage subsidised by the government translates into a continuous rise of the tax level and of the reservation product for stable jobs and the transformation threshold for subsidised jobs whereas these variables start to decrease before increasing when firing costs are low (they reach a minimum around $\sigma = 0.4$) (see figure 3). Whatever the level of firing costs, unemployment decreases and regular employment rises but the efficiency of a higher subsidy is limited to a given threshold (corresponding for regular employment
This threshold is higher for unemployment as it keeps falling even when regular employment decreases, this reduction being offset by a rise in subsidised employment ($\sigma = 0.65$ or $\sigma = 0.7$ according to the level of firing costs). The consequences on employment are more favourable in a flexible labour market as the tax and the various reservation productivities first fall, the policy being self-financed\footnote{In such a market, when $\sigma$ starts to increase, the expected value of a subsidised job $JS$ first slightly decreases before rising (when the subsidy value is small, it affects more firing costs than wage costs (as the firm has to pay back the subsidy received if it fires the worker)). It brings about a shift downward of the job creation curve and the various reservation productivities fall, the net effect being a decrease in the tax rate.}. The differences in results compared to the previous models of Mortensen [1996] and Mortensen and Pissarides [2001] partly lie in the fact that firms are not able to transfer to workers a part of the subsidy burden through a decrease in the bargained wage and are thus led to adjust their labour demand, some workers becoming too costly (they actually raise their reservation products for all jobs). The employment consequences of the subsidy are thus less favourable. Moreover, these authors only considered an increase in the amount of the subsidy and did not study the effect of other policy parameters which negatively act upon employment.

Figure 2: A rise in the proportion of subsidised jobs in the case of a tax financing (— low firing costs and - - - high firing costs) respectively to $\sigma = 0.55$ and $\sigma = 0.6$ in the case of high and low firing costs).
Figure 3: An increase in the subsidy with a tax financing
Finally, if the government decides to lengthen the duration of subsidised jobs (from six months to two years), the reservation productivity of the firms for these jobs, $R_s$, falls whatever the level of firing costs. The continuing job value gets higher, the subsidised job being more advantageous as the subsidy is paid during a longer period. Nevertheless, with the increase in contract duration, the policy becomes more expensive thus entailing a rise in the reservation productivities of the various types of jobs (permanent jobs and ending subsidy jobs). This taxation effect more than offsets the positive employment effect of a rise of subsidised jobs duration. The latter effect is more pronounced when firing costs are high as the shifts in the job creation and the job destruction curves are greater (for a proof see Appendix 2). Whatever the level of firing costs, the unemployment effect of such an increase in the rate out of subsidised jobs is positive and the regular employment effect is negative but their magnitude is increased by labour market flexibility (see figure 4).

5.2 The case of a replacement rate financing

We then focus on the case of a policy financing through a reduction in the replacement rate of unemployment benefits. This option comes within the scope of the general setting of a labour market reform as advocated by the OECD. Several countries have adopted some measures to "activate" their passive spending to favour a recovery of employment growth. In this setting, the subsidy policies are efficient to reduce the unemployment rate of less-skilled workers and their efficiency is strengthened by a rigid labour market. In the case of a self-financed policy, we will limit the increase in the replacement rate to 25% to maintain work incentives.
These results may be explained as follows. Let first consider the case of a rise of the share of subsidised hires. If firing costs are high, it brings about a strong rise in the level of regular employment (a rise of 0.7 percentage points if the latter reaches 0.12 of the jobs created (when the replacement rate has its binding value) (see figure 5). However, the efficiency of such a policy is limited to a given threshold. Above this threshold, a rise in $\gamma$ translates into regular employment reaching an upper limit (an increase of 7 percentage points, the unemployment rate falling of 10 percentage points when $\gamma$ equals 0.4 for high firing costs). This effect is less pronounced in the case of low firing costs, regular employment being maximum when $\gamma = 0.25$ and the unemployment rate reducing of 3 percentage points. In the case of a replacement rate financing, the negative tax effect on the various reservation products does no longer exist as the unemployed workers bear the cost of employment policy through a reduced replacement rate. The crowding-out effect of the policy disappears. Moreover, this policy does not diminish the job seekers’ welfare whatever the level of firing costs because it substantially reduces the unemployment rate. The decrease in spending which occurs is sufficient to preserve and even increase the amount of unemployment benefits. Firms create more jobs because the mean reservation product in the economy falls following the increase in the number of subsidised jobs. The policy induces more job openings in a flexible labour market (see Appendix 2). Nevertheless, its effects are more favourable on a rigid labour market as job destructions are less numerous, the various reservation productivities being lower when firing costs are high.

If the government decides to favour less skilled job seekers hires by raising the amount of the subsidy, it entails a decrease in the unemployment rate (it respectively falls by 2, 8 and 1 percentage points whether firing costs are high or low). It is worth noting that if the subsidy policy is adopted, the reduction in the unemployment level is not only due to the increase in subsidised employment, regular employment also rises (see figure 6). This result stems from the fact that the cost of a vacant job decreases with the employment subsidy and that among subsidised jobs, some will become regular jobs after the subsidy period. The subsidy effect is thus not limited to a windfall effect as some workers are retained in their jobs at the end of the subsidy programme. It is widened by

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12 When $\gamma$ is raised, the reservation productivities of all jobs remain unchanged. Nevertheless, the policy implementation allows less skilled workers to find out a job as subsidised jobs become more numerous and the reservation productivity $R_s$ for these jobs is lower than the productivity associated to the continuation of a regular job, $R_d$. 

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Figure 5: The employment effects of an increase in the proportion of firms benefiting from the subsidy with a replacement rate financing
labour market rigidity. An increase in the subsidy value generates a decrease in the reservation productivity of the firms benefiting from subsidised jobs and consequently a fall in the unemployment rate. This fall is more pronounced when firing restrictions are larger as the reservation productivity necessary to turn a subsidised job into a stable one and the reservation productivity for stable jobs are lower. The mean reservation productivity in the labour market being lower, the job destruction rate is reduced and the preserved matches are more numerous. The more rigid is the labour market, the more the replacement rate of unemployment benefit rises as the policy is more cost-effective (see figure 6).

Finally, a decrease in the rate at which the policymaker ends the programme leads to a rise in the subsidised job profitability. The subsidy will be received

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\[\text{Figure 6: A rise in the level of employment subsidy in the case of a replacement rate financing}\]

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\[\text{13 If firing costs are low, the unemployment rate starts to increase before falling when } \sigma \text{ gets higher. This rise is due to the decrease in } JS \text{ as we saw in footnote (13). The employment effect is less favourable in the case of a replacement rate financing as the negative effect of taxes on the various reservation productivities does no longer exist.}\]

\[\text{14 The scope of the job destruction and creation curves do not depend on firing costs, so the shifts of these curves are similar (see Appendix 2).}\]
Figure 7: An increase in the duration of the subsidy payment when the policy is financed by the replacement rate

during a longer period and thus firms will be less demanding as far as productivity is concerned. The subsidised job destruction rate then decreases as does the unemployment rate, this reduction being more important in a rigid labour market (2.5 percentage points against 0.9). However, the effect on stable employment varies with labour market rigidity: it is positive when firing costs are high whereas it is negative in the opposite case (see figure 7). We may interpret the latter effect as follows: when the level of firing costs is low, the decrease in the reservation productivity for subsidised job $R_s$ does neither favour job creations nor limit job destructions enough and there is a crowding-out of regular employment by subsidised employment. As we saw it earlier, the employment consequences of programme’s lengthening are better in the case of a rigid labour market. The replacement rate for unemployment benefit increases in both cases but its raise is higher when job protection is strong.

6 Conclusion

In this paper, we show that the institutional setting in which the policy is implemented is a crucial element for its efficiency. Employment subsidies in a rigid labour market do not have the same consequences as in a flexible one. If some conditions are fulfilled, this policy may constitute a useful tool to fight unemployment and, particularly, the adverse consequences of long term unemployment as it offers the opportunity for less productive workers to get a job on the regular labour market. Thus, instead of removing all rigidities on the labour market which may bring some welfare to workers, the government should choose to combine employment subsidies with other labour market institutions such to avoid their negative effects. According to Gregg and Manning [1997], firing costs actually give workers some power to offset the monopsony power of employers on the labour market. These costs also represent an efficient mechanism of insurance for workers (for a recent analysis in a matching setting see Pissarides...
and above all decrease job destructions (see the various matching models of Mortensen and Pissarides).

The qualitative conclusions of the model confirm the fact that each policy parameter matters as it acts differently upon labour market equilibrium. If the government increases the number of subsidised jobs, it entails a shift of the job creation curve downwards, the creations of subsidised jobs being less costly than those of stable jobs. In the case of a financing through payroll taxes, an additional effect appears. The stable job destruction curve and the transformation curve move upwards, the financing of employment subsidies leading to an increase in the taxes borne by firms. Wages cannot adjust to limit this effect as a minimum wage prevails in the economy. The effect of a rise in the amount of the subsidy differs, it also shifts downwards the job destruction curve for subsidised jobs. The effect arising on the other productivities is the same as previously, in the case of a tax financing. Finally, a lengthening of the duration of the period of subsidy payment brings about a fall in the reservation productivity of the firms benefiting from the subsidy. It affects both the job creation and the job destruction curves. The analytical results being indeterminate, we have investigated the net effects thanks to a simulation including some features of the French labour market.

The main results of the model’s simulation are the following. We have shown that financing employment policy through increases in payroll taxes may have low positive or even adverse effects on employment either in the case of a rise in the number of vacancies benefiting from a subsidy or in the amount of this subsidy. The level of firing cost affects the policy efficiency. Its impact is more favourable in a flexible labour market but is nevertheless limited. Shortening the duration of the period during which the subsidy is received by firms is then efficient as the policy does not succeed in reducing unemployment. These policy negative effects may be attributed to the existence of a minimum wage which prevent wages from adjusting to the rise in taxes. This wage rigidity and the inclusion of the various policy parameters explain the differences with the results obtained by previous models which were only considering an increase in the subsidy amount. Here, the rise in reservation products is so large that it more than offsets the positive effects of the subsidy on job creations. The only case where the policy may be efficient is when the government rises the amount of the subsidy and when firing costs are low. These results may partly contribute to explain the reduced efficiency of subsidies implemented in some European countries (for recent surveys, see Fay [1996], Meager and Evans [1998], and for France, the studies of Kramarz and Philipp [2001] and Gélot [1997]).

If the government decides to finance employment policy through reductions in the replacement rate - in accordance with the OECD recommendations concerning the activation of passive spending - the results are far more encouraging. The efficiency of employment subsidies is raised by labour market rigidity. The unemployment rate is reduced and the stable employment rate increases before

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reaching a maximum when the share of subsidised jobs is raised for both levels of firing costs. If the government chooses to make the amount of the subsidy higher to induce firms to hire low productivity job seekers, stable employment increases and unemployment decreases in a rigid labour market but they are respectively U-shaped and hump-shaped in the case of low firing costs. The net effect of the employment policy is not reduced to a pure windfall effect, because workers may keep their jobs at the end of the subsidy period if they have reached a sufficient productivity. Lengthening the duration of the subsidy payment is also efficient to reduce unemployment. However, when firing costs are low, this decrease is reached through a fall in regular employment, subsidised employment crowding-out the former.

This model emphasizes that each characteristic of the employment policy plays its role as it acts through different channels: an increase in the share of job seekers benefitting from the subsidy has a sizeable impact on unemployment through its action on job creations but a raise in the subsidy level substantially modifies the behaviour of firms as it decreases their reservation productivity. Finally, lengthening the period during which firms perceive the subsidy simultaneously affect their hiring and firing decision but its marginal employment effect is less important. The next step to this analysis is to differentiate the consequences of this policy according to the duration of unemployment spells experienced by job seekers to account for the fact that negative duration dependence may be limited thanks to the training option of subsidy programme, which helps to restore their productivity.

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7 Appendices

7.1 Appendix 1

The reservation productivity for stable jobs is higher than the reservation productivity for subsidised job for suitable values of the subsidy, its duration and the firing costs. Using equations (10), (11) and (14), the demonstration is the following:

\[
R_d - R_s = \frac{\delta + \mu + \lambda}{\delta + \mu + \lambda \Phi(R_0)} \left[ \frac{\lambda R_0}{\lambda + \delta + \mu} + \frac{1}{R_s} \int_R (x - R_s) d\Phi(x) + \frac{1}{\lambda + \delta} \int_R (x - 1) d\Phi(x) \right] + \frac{\lambda(1 - \Phi(R_0))}{\delta + \mu + \lambda \Phi(R_0)} \left[ 1 - w(1 + \tau) \right] - \frac{\lambda(1 - \Phi(R_0))}{\delta + \mu + \lambda \Phi(R_0)} \left[ 1 - R_d \right] + \frac{\delta}{\delta + \lambda} \int_{R_d} \Phi(x) dx + T \left[ \frac{\delta + \mu + \lambda}{\delta + \mu + \lambda \Phi(R_0)} - \delta \right]
\]
This expression is positive if the last term is positive which is the case for plausible values of the firing costs for subsidised jobs ($\zeta$) and if the following condition is fulfilled (sufficient condition) : $(\delta + \mu + \lambda) \frac{\sigma}{\lambda} - \lambda[\delta + (\mu + \lambda)\Phi(R_0)] \frac{1-R_s}{\lambda} > 0$ which is the case if $\frac{\sigma}{\lambda} > 1 - R_d$ (the amount of the subsidy has to be large enough).

The job creation condition states a decreasing relationship between the reservation productivity for stable jobs and labour market tightness. The derivative of labour market tightness is defined as: $\frac{d\theta}{dR_d} = \frac{-\frac{\psi_r}{\psi_{\sigma}} - \frac{\psi_{R_d}}{\psi_{\sigma}}}{\frac{\psi_{\sigma}}{\psi_{R_d}} \frac{dR_d}{dR_s}}$. 

Using equations (6), (7), (10), (11), (12), (13), (14) and (15) this expression becomes:

$$\frac{d\theta}{dR_d} = \frac{\left(\frac{1}{\lambda} + \frac{\gamma R_0}{\sigma + \lambda + \lambda \Phi(R_0)} \right) + \frac{\gamma \rho}{\sigma + \lambda + \lambda \Phi(R_0)} \left(1 + \frac{R_s}{\lambda} - T\right)}{h'(\theta) k}$$

This expression is negative as the hiring rate decreases with labour market tightness ($h'(\theta) < 0$) and the expression $\frac{1-R_s}{\lambda} - T$ equals $J_d(1)$ which is positive. The job creation condition thus exhibits a negative slope in the $(\theta - R_d)$ space.

### 7.2 Appendix 2

The various partial derivatives of the job creation and the job destruction conditions are the following (using equations (11), (14)) and (15)):

$$\frac{\partial JC}{\partial \gamma} = -\frac{\psi_r}{\psi_{\sigma}} = \frac{J_s - J_d}{h'(\theta) k}$$

$\frac{h'(\theta)}{h(\theta)^2}$ being negative, the sign of the derivative depends on the differential between the expected value of a subsidised job and of a regular one. This differential is positive otherwise the subsidy would not be efficient in favouring job creations (moreover, we have shown in the preceding appendix that $R_s < R_d$).

The efficiency of a rise of the proportion of subsidised jobs depends on the level of firing costs : $\frac{\partial JC}{\partial \sigma} = \frac{\gamma \Phi(R_0)}{\lambda} \frac{1+\Phi(R_0)}{\lambda} > 0$. Thus, the effect of an increase in $\gamma$ on the job creation curve decreases with labour market rigidity.

The job destruction condition for a subsidised job does not depend on the proportion of subsidised jobs ($\gamma$).

$$\frac{\partial JC}{\partial \sigma} = -\frac{\psi_r}{\psi_{\sigma}} = -\frac{\gamma \Phi(R_0)}{\lambda} \frac{h'(\theta) k}{h(\theta)^2} \text{ and } \frac{\partial JD_s}{\partial \sigma} = -\frac{\Gamma_s}{\Gamma_{R_s}} = \frac{\delta + \mu + \lambda}{\delta + \mu + \lambda \Phi(R_0)}$$

The first derivative is positive whereas the second one is negative, so the job creation condition moves upwards and the subsidised job destruction condition, downwards.
\[
\frac{\partial JC}{\partial \mu} = -\frac{\Psi_{\mu}}{\Psi_{\theta}} = \frac{\gamma}{\delta + \mu + \lambda \Phi(R_0)} \left[ (1 - \Phi(R_0)) J_{SH}(1) - J_d(1) \right] + \frac{1}{\lambda + \delta + \mu} \int_{R_s}^{R_0} (x - R_s) d\Phi(x)
\]

The job creation curve moves downwards when \(\mu\) increases as \(J_{SH}(1) - J_d(1) > 0\) with \(J_d(1) = \frac{1}{\delta + \lambda} - T\) (the value of a high productivity subsidised job starting with the higher level of productivity is greater than the value of a stable job as the subsidy policy is efficient to induce job creations by firms).

\[
\frac{\partial JD_s}{\partial \mu} = -\frac{\Gamma_{\mu}}{\Gamma_{R_s}} = \frac{\lambda (1 - \Phi(R_0)) J_{SH}(1) - J_d(1)}{\delta + \mu + \lambda \Phi(R_0)} - \frac{\lambda}{\lambda + \delta + \mu} \int (x - R_s) d\Phi(x)
\]

This derivative is positive and the subsidised jobs destructions rise when \(\mu\) is raised by the government. But we are interested in a lengthening of the period during which the latter pays the subsidy, so the sign of the derivatives are the opposite when \(\mu\) falls. The effect of a reduction in the duration of subsidy payment depends on the level of firing costs, the two derivatives being increasing functions of firing costs:

\[
\frac{\partial JC}{\partial \mu} = \frac{\gamma (1 - \Phi(R_0)) [J_{SH}(1) - J_d(1)] + \frac{\lambda}{\lambda + \delta + \mu} \int (x - R_s) d\Phi(x)}{\delta + \mu + \lambda \Phi(R_0)}
\]

If the employment subsidy is financed by payroll taxes, a rise in each policy parameter entails an increase in the tax level. It affects the various job creation and destruction conditions the following way:

\[
\frac{\partial JC}{\partial \tau} = -\frac{\partial JC}{\partial \sigma} < 0, \quad \frac{\partial JD_s}{\partial \tau} = -\frac{\partial JD_s}{\partial \sigma} > 0, \quad \frac{\partial JD_d}{\partial \tau} = \frac{(\delta + \lambda) \pi}{\delta + \mu + \lambda \Phi(R_d)} > 0 \quad \text{and} \quad \frac{\partial JD_0}{\partial \tau} = \frac{\partial JD_d}{\partial \tau} > 0
\]

References


