Insurance and Financial Hedging of Oil Pollution Risks¹

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Abstract

The current international regime that regulates the maritime oil transport calls for financial contributions by oil firms once an oil spill has occurred. Their percentage of contribution to the International Oil Pollution Compensation Fund does only depend on their level of activity. In this paper, we show that this compensation regime would be more efficient if contributing oil companies adopt financial strategies relying on hedging oil pollution risks. Indeed, the optimal coverage contract is such that standard insurance is useful to manage small and medium oil spills, while investments on financial markets help to cover huge oil spills, less frequent but much more catastrophic for Society. We also show that prevention of oil spills increase when insurance is bundled with a financing hedging strategy. This positive effect on prevention is still enhanced when firms have the opportunity to send signals about their risk-reducing activities to potential investors.

Key-Words: oil spill, legislation, insurance, capital markets, prevention, catastrophe.

JEL Classification: D80, G22, Q25.
1 Introduction

The maritime transport of oil is regulated by the 1992 Civil Liability Convention in most countries of the world\textsuperscript{1}, except mainly for the United States, which have their own Convention\textsuperscript{2}. In this paper, we focus on the compensation system implemented when an oil spill is registered in the territorial sea of any member of the 1992 Civil Liability Convention. Since oil spills can create severe damages to the environment but also to the human activities near the coast, they may induce huge claims, which cannot be covered without a compensation system adapted to those catastrophic losses. Hence, we will show that the current International regime would benefit from a reorganization involving both standard insurance and financial hedging.

The 1992 International Oil Pollution Compensation Fund (1992 IOPC Fund) participates in the compensation of victims of an oil spill if the payment already granted by the insurer of the owner of the tanker\textsuperscript{3} is not sufficient. The contributions of oil firms to the Fund are proportional to the quantity of oil received in a year and they are due each time an oil spill has occurred in the territorial waters of a member, whatever the flag of the tanker and whatever the citizenship of the oil firm. Hence the IOPC Fund enables to compensate victims even if the owner of the tanker is not a citizen of a member state and empirical facts show that the IOPC Fund seems to be rather efficient in minimizing the time between the oil spill event and the effective compensation of victims. However, funds are levied at random dates and expenses are not smoothed through time. Hence we will show that within the current international regime, oil firms would benefit from capital markets by resorting to appropriate financial instruments. Financial mechanisms

\textsuperscript{1}81 states ratified the 1992 Civil Liability Convention on 20 November 2002.
\textsuperscript{3}Under the 1992 Civil Liability Convention, only the owner of the tanker is held financially liable for the catastrophe. The convention obliges him to buy pollution insurance, provided by P&I Clubs which are non-profit making mutual insurance associations. These mutual groups offer insurance depending on the size of the boat and not directly on the damages that may be induced by a wreck. Hence, insurance may be limited, as it was the case for the Erika’s wreck on the French coast in December 1999 (7\% of the total available funds).
improve and complete the hedging provided by insurance policies which prove to be insufficient to cover alone huge risks related to oil spills.

The oil industry has several interests in using hedging mechanisms to make the current compensation regime more efficient. Without adequate risk management, oil firms lose some efficiency in their activities and the cost induced by this inefficiency is lost for the victims’ compensation. In particular, the 1992 IOPC Fund, as it works, does not rest on the risk transfer principle. By defining contributions on the basis of the aggregate risk of the pool, the mutuality principle (Borch (1962), Wilson (1968)) is applied. Nevertheless, the aggregate risk is still variable because of the possible huge consequences of an incident and because of the limited number of contributing members in the Fund. Consequently, the mutuality principle is no longer sufficient to spread all the risk on the oil firms. We show that capital markets seem to be able to solve the issue of diversification and also to mitigate transaction costs. Doherty (2000) gives several arguments that make insurance profitable for firms, and that enhance the fact that insurance mechanisms have to be completed by some investment on capital markets when dealing with large risks. Froot (2001) provides also different reasons why markets are more efficient than insurers in global risk reductions. One important point is that securitization may reduce transaction costs such as administrative fees or costs related to agency issues. In the same spirit as in Doherty and Dionne (1993), Schlesinger (1999), Doherty and Schlesinger (2002) and in Mahul (2002), we show that insurance bundled with a financial hedging strategy dominates a situation with only standard insurance. However, our economic context is rather different from these studies. In our framework, each individual bears a percentage of the aggregate risk of the pool (here, the IOPC Fund) and an individual risk of bad reputation that is positively correlated to the aggregate risk and non insurable. Up to now, the litterature focused essentially on risks that can be split into idiosyncratic risk,

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4A large part of total contribution is done by a small number of oil companies. This can be explained by the concentration of the oil sector and the exoneration of contributions of companies belonging to member states receiving less than 150,000 tons of oil a year. Indeed, the application of the mutuality principle, which rests on the law of large numbers, is less effective in this context.
specific to the individual and easily insurable, and a systematic risk, independent from the idiosyncratic one. Losses induced by reputation constitute a central variable in our model; reputation and its impact on firms’ value has become a major concern for firms involved in environmentally risky activities for the environment as shown by Lanoie et al. (1998). This issue is even more crucial for oil companies in the wake of an huge oil spill. Another important point of our analysis deals with prevention, which is not considered in the previous models. We show that financial hedging may give additionnal incentives to oil firms to invest in prevention. This result is important when focusing on the current discussions that are held at the European Commission about the evolution of the financing of the IOPC Fund. Especially, it is argued that an increase of individual contributions may improve the safeness of boats chartered by oil firms and allows it to fully compensate victims of infrequent huge oil spills. Our main aim in this paper is different. We will show that prevention against maritime oil pollution becomes more valuable if oil firms apply adequate financial strategies. These strategies imply to hedge oil spill incidents by resorting simultaneously to insurance policies and investment on capital markets.

The paper is organized as follows. The second section focuses on the current regime of the IOPC Fund. In the third section, we present the basis model and we introduce standard insurance mechanisms in order to define the optimal insurance contract an oil firm (or the Fund) can buy to an insurer. It entails a deductible with coinsurance for all losses higher than the deductible. In the fourth section, we show that financial hedging may be a good way to cover the residual risk still retained by oil firms (or by the pool) after insurance. When incorporating this point in the insurance contract, the risk premium asked by the insurer decreases and more standard insurance becomes

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5 In May 2003, under the auspices of the International Maritime Organization, a protocol which introduces a third tier of compensation has been adopted. Its aim is to increase significantly levels of compensation if compensation available through the Civil Liability Convention and the IOPC Fund should prove to be insufficient. Note that the signature of this new protocol is not compulsory and may de facto exclude poorer countries because of the high levels of contribution in case of an incident.
available for small and medium oil spills, while capital markets are useful for hedging huge oil spills. Another important point is that financial markets may provide incentives to invest in prevention by allowing firms to give positive signals to potential investors. Section five concludes the paper and discusses the implications of our results. All proofs are given in Appendix.

2 The IOPC Fund and risk mutualization

In this section we present the main features of the current legislation and we provide some insights about how the current financing of the IOPC Fund can be improved. The issues we focus on are formalized in sections 3 and 4.

Since 24 May 2002, International maritime transport (except for the United States) is exclusively regulated by the 1992 Civil Liability Convention (CLC in the course) and by the 1992 International Oil Pollution Compensation Fund (IOPC Fund) Convention. The Fund is financed by contributions of the oil industry of member states receiving more than 150,000 tons of oil per year after sea transport. The contribution of each company is proportional to the annual tonnage received by sea and is directly payable to the Fund. Contributions, decided each year by the Assembly of the Fund, cover administrative costs and estimated compensation payments for passed pollutions. Hence no provision is made ex ante and each oil firm pays an ex post indemnity equal to a percentage of the losses induced by all oil spills registered until this date: the sum of these oil spills can be considered as the aggregate loss of the IOPC Fund, which is shared among its members.

This International compensation regime seems to be rather efficient: It has improved the protection of sea environment against oil pollution by inducing a decrease of the number of huge oil spills in the last two decades. Also, it facilitates claims settlement

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6 Actually, the first Civil Liability Convention is dated from 1969 and the Fund was created in 1971. Both were amended in 1992. For details, see the companion paper of Schmitt and Spaeter (2003).

7 The number of large oil spills (spilling more than 700 tons) was 7.3 per year on average during
for victims of pollution and it has increased compensation available for them. Although claims for damage to the ecosystem are not admissible, compensation is granted to a wide range of costs (clean-up operations, property damages, economic losses, ...). Besides, contributions borne by the oil industry are fair compared to the revenues induced by oil activities\(^8\).

Nevertheless this regime also shows its limits regarding the total compensation available for victims\(^9\) and the incentives to enhance environmental prevention through the chartering of safety boats. Indeed, while the shipowner is solely held liable through the Civil Liability Convention, the whole oil industry participates in compensations through the IOPC Fund Convention: no direct compensation between the owner of the oil escaped from the boat and victims can be established. From a theoretical point of view, Ringleb and Wiggins (1990) show that such considerations may lead firms to subcontract risky activities, here the maritime transport of oil, in order to escape from prosecution in the case of an accident. Moreover, the contribution to the Fund is upper bounded and this kind of limited liability may induce oil firms to charter boats with medium, or even weak, levels of quality.

Besides, the shipowner is also protected by limited liability, which benefits mostly low market value firms as shown by Schmitt and Spaeter (2002). Consequently, risk-reducing activities may still be worsened.

\(^8\)From 1996 to 2001, the annual contributions represented at most 0,05% of the price per ton of crude oil received.

\(^9\)Only partial compensation was available to victims after the wrecks of Nakhodka (1997), Erika (1999) and Prestige (2002). In the case of Erika, the percentage of compensation was the highest one among these three catastrophes: About 80% of the total losses estimated by the experts of the IOPC Fund. Concerning the Prestige incident, the Executive Committee of the IOPC Fund decided in May 2003 to limit compensation to 15% of the loss actually suffered by the respective claimants. On 31th of March 2004, this level of payments was maintained.

\(^1\)The 1990s compared to 24.2 during the 1970s. (source : ITOPF Handbook 2003-2004) However, the level of losses eligible for compensation has increased dramatically in some huge incidents. This can be explained by the higher than average increase of population in coastal areas and the development of tourism.
3 Basis model and insurance

First, we consider the current situation of the IOPC Fund, where no insurance is available. Then we introduce insurance mechanisms.

3.1 (Catastrophe) Risk mutualization

Consider \( n \) oil firms located in states that are members of the Fund. We denote \( \tilde{x}_i \) the risk of loss borne by Society when Firm \( i \) charters a given boat for the transport of its oil. This random loss \( \tilde{x}_i \) takes the strictly positive value \( x_i \) with probability \( p_i \) and equals zero with probability \( (1 - p_i) \). Probability \( p_i \) of incident is affected by the level of prevention \( e_i \) decided by the oil firm that means, here, by the safeness of the chartered boat: \( p_i = p(e_i) \) with \( p'(e_i) < 0 \). The cost of prevention is defined as \( c(e_i) = e_i \); \( e_i \) increases as the safeness of the ship chartered by the oil firm increases. We suppose that the charterer, i.e. the oil firm, controls the quality of the boat and the competence of the crew\(^\text{10}\). Finally the aggregate risk of the Fund is \( \tilde{X} = \sum_{i=1}^{n} \tilde{x}_i \) with values\(^\text{11}\) in \([0, L]\) and with distribution function \( F(X/e) \), where \( e \) is the vector of all individual investments in prevention: \( e \equiv (e_1, \ldots, e_n) \). An increase in the level of individual prevention of, at least, one firm improves the distribution in the sense of the first order stochastic dominance, but at a decreasing rate: \( F_{e_i} > 0, F_{e_i e_i} \leq 0, \forall X \in [0, L] \) and \( F_{e_i}(0/e) = F_{e_i}(L/e) = 0 \).

As it works currently, each time an accident is registered in the territorial waters of its members, the Fund calls for contributions by each oil firm. The percentage of contribution of Firm \( i \) denoted \( \alpha_i \) is applied to the level of the aggregate loss \( X \) of the pool, up to a maximum value \( \widehat{X} \), which is assumed to be less than the amount of losses registered if all chartered boats would have an accident in the same period\(^\text{12}\): \( \widehat{X} < L \).

\(^{10}\)This may look as a bold hypothesis knowing that the maritime oil transport is largely subcontracted to shipowners. However, charterers get a precise information on the safeness of a boat through the classification society used to check it.

\(^{11}\)Here, \( L \) is simply equal to the sum of the strictly positive values of all oil spill variables: \( L = \sum_{i=1}^{n} x_i \). We assume that \( n \) is sufficiently large so as to consider \( \tilde{X} \) as a continuous variable.

\(^{12}\)This is assumed to reflect the fact that the IOPC Fund, as it works, is not able to compensate for...
The amount of money available from the IOPC Fund is bounded so that firms benefit from a kind of limited liability.

In this system, which characteristics are similar to the ones of the mutuality principle, a given firm does not bear all the risk directly linked to the boat she charters since it is spread across all members of the Fund.

In addition of the risk $\alpha_i \tilde{X}$, firms have to bear a second risk related to (bad) reputation. Indeed, each time an oil spill occurs, the whole industry is affected by the harsh opinion of the public. Nevertheless, the effect of bad reputation is stronger for the firm that owned the spilled oil because of the bad advertising which is made around her brand. Finally, each firm bears a bad reputational effect composed of an individual effect, which is zero if the firm is not implied in the wreck, and a general effect which is positive for any incident\(^{13}\). Formally, the random variable describing the total reputational effect is denoted $-g(\tilde{X}, \tilde{x}_i)$ with $0 < g_X < g_{x_i}$ and $g_{XX} < 0$. The preferences of the firm are represented by a Von Neumann Morgenstern utility function $u(.)$ and she owns an initial non random wealth $w_i$.

In the current functioning of the IOPC Fund, the oil firm can only choose the level of quality $e_i$ of the boat to be chartered. She chooses it in order to maximize her expected net welfare:

$$\max_{e_i} R = \int_0^{\tilde{X}} (u(w_i - \alpha_i X - g(X, \tilde{x}_i)) - e_i) f(X/e) dX$$

$$+ \int_{\tilde{X}}^L (u(w_i - \alpha_i \tilde{X} - g(X, \tilde{x}_i)) - e_i) f(X/e) dX, \quad (1)$$

where $g(X, \tilde{x}_i)$ is the expected value of the reputational effect evaluated with respect to all oil spills.

\(^{13}\)Both effects are not specific to the oil industry. ?? (1987) obtain similar conclusions by studying empirically the impact of the Bhopal incident in India (1984) on the stock value of chemical firms in general and on Union Carbide India Limited (UCIL), which was responsible of the pesticides leak, in particular. See also (Freedman and Stagliano, 1991) who show that firms with a high level of disclosure about their risk-reducing activities suffer from a smaller decrease in their stock price after an incident.
\( \bar{x}_i: \)

\[
g(X, \bar{x}_i) = p(e_i)g(X, x_i) + (1 - p(e_i))g(X, 0), \quad \forall X \in [0, L] \tag{2}
\]

In the course of the text, we adopt the following notations: \( w_f = w_i - \alpha_i x - g(X, \bar{x}_i) \), \( \bar{w}_f = w_i - \alpha_i \bar{x} - g(X, \bar{x}_i) \), \( g_{e_i} = g_{e_i}(X, \bar{x}_i) \) and \( g_X = g_X(X, \bar{x}_i) \). In Proposition 1, we discuss the impact of a variation of the upper bound \( \bar{X} \) of the pool on the willingness of firms to charter safer boats.

**Proposition 1**

i) For given prevention levels of the other oil firms, the optimal level of prevention \( e_i^* \) of Firm \( i \) satisfies the following first order condition:

\[
1 = - \int_0^{\bar{X}} g_{e_i} u'(w_f) f(X/e) dX - \int_{\bar{X}}^L g_{e_i} u'(\bar{w}_f) f(X/e) dX
+ \int_0^{\bar{X}} (\alpha_i + g_X) u'(w_f) F_{e_i}(X/e) dX + \int_{\bar{X}}^L g_X u'(\bar{w}_f) F_{e_i}(X/e) dX \tag{3}
\]

ii) An increase of the upper bound \( \bar{X} \) of the funds available for clean-up and compensation through the pool induces an increase in the level of prevention, other things being equal.

The left term of Equality (3) is the expected marginal cost of prevention. From our assumptions, this amount is certain and equal to one. The right-hand-side term is the expected marginal benefit of prevention. First, chartering safer boats will reduce the risk of bad reputation (first and second term) because the probability for Firm \( i \) to be directly involved in an accident (probability \( p_i \)) decreases as \( e_i \) increases. Second, prevention has also a positive impact on the aggregate risk of the Fund since it improves its distribution. Firm \( i \) will benefit from an additionnal reduction in the bad reputation due, this time, to the reduction of the aggregate risk of the pool (third and fourth term). Lastly, the presence of \( \alpha_i \) in the third member of the right-hand-side term represents the direct benefit of prevention: increasing prevention reduces the risk \( \alpha_i \bar{X} \) borne by Firm \( i \).
This first order condition will be useful in the course for comparing the different levels of prevention obtained when the firm will have successively access to, first, standard insurance and, second, to a joint contract that displays standard insurance and financial hedging.

With Point ii) of Proposition 1, we are able to analyze the impact on the firms' behavior of a change in the upper limit $\hat{X}$ of the Fund. From an empirical point of view, the cap of the IOPC Fund were increased just after the Erika incident, and two more times during the year 2003. In our theoretical framework, an increase of $\hat{X}$ yields an increase in the level of prevention by Firm $i$, other things being equal. Having to pay more for large accident is similar for the firm to bearing more risk. Thus the marginal benefit of prevention increases, while the monetary marginal cost of prevention remains unchanged: The price of chartering safety boats is not affected. Finally, the oil firm has incentives to increase the level of preventive investment $e_i$.

Hence increasing the maximum level of contribution by oil firms to the Fund may be a good way to increase both the available funds in the case of an oil spill and ex ante prevention. However, this fragilizes the risk mutuality effect since more aggregate loss is borne by each individual firm. Furthermore, small firms may have some difficulties to fulfill their commitments if their contributions become too high.

In what follows, we focus on standard insurance as a manner to increase contributions to the Fund. The idea is that firms may be able to contribute more to the Fund if their random contributions are insured. Standard insurance can be bought by each individual firm or by the pool. Here we choose the first alternative.

### 3.2 Optimal standard insurance contract

Now, assume that the oil firm can transfer a part or the whole of her risk $\alpha_iX$ to an insurer. The compensation function is denoted $C(\alpha_iX)$. The insurance premium $Q$ is equal to the expected costs and indemnities the insurer will have to take in charge: $Q = (1+\lambda)E[C(\alpha_iX)]$ where $\lambda$ represents the administrative costs of the insurer plus the
risk premium per unit of transferred risk and $E$ the expectation operator over $X$. The Von Neumann Morgenstern utility function of the insurer is denoted $v(.)$ with $v'(.) > 0$ and $v''(.) \leq 0$ and $W$ is his initial wealth. The compensation function $C(.)$ is defined over $[0, \alpha_i L]$.

The maximization program of the oil firm subject to the participation constraint of the insurer becomes:

$$
\max_{C(.)} R_C = \int_0^L \left( u(w_i - \alpha_i X 1_{\{X \leq \bar{X}\}} - \alpha_i \bar{X} 1_{\{X > \bar{X}\}} + C(\alpha_i X) - Q - g(X, \bar{x}_i)) - \epsilon_i \right) f(X/e) dX \\
\text{subject to } \int_0^L v(W + Q - (1 + \lambda)C(\alpha_i X)) f(X/e) dX \geq v(W)
$$

We use optimal control to solve this maximization program. The random variable $X$ plays the role of time, $C(.)$ is the control variable while the state variable is $z(X) = \int_0^X v(W + Q - (1 + \lambda)C(\alpha_i t)) f(t/e) dX$. Its evolution is described by the system:

$$
\begin{align*}
\dot{z}(X) &= v(W + Q - (1 + \lambda)C(\alpha_i X)) f(X/e) \\
z(0) &= 0 \\
z(L) &= v(W)
\end{align*}
$$

The Hamiltonian of Program (4) is

$$
H(X) = \left( u(w(f(X, \bar{X})) - \epsilon_i + \mu(X) v(W + Q - (1 + \lambda)C(\alpha_i X))) \right) f(X/e), \quad (5)
$$

with $w(f(X, \bar{X})) = w_i - \alpha_i X 1_{\{X \leq \bar{X}\}} - \alpha_i \bar{X} 1_{\{X > \bar{X}\}} + C(\alpha_i X) - Q - g(X, \bar{x}_i)$ and $\mu$ the Lagrange function. The contract $C^*$ that maximizes $H$ is presented in Proposition 2 hereafter.

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14Function $1_{(.)}$ is the indicator function, taking value one if the condition into brackets is satisfied, zero otherwise.
Proposition 2

(i) The optimal insurance contract displays a positive deductible when administrative costs are increasing in the level of indemnities. Marginal compensations for damages beyond the level of deductible but lower than \( \hat{X} \) are given by

\[
C''(\alpha_i X) = \frac{\left(1 + \frac{g_X}{\alpha_i}\right) R_u}{R_u + (1 + \lambda) R_v},
\]

with \( R_u \) and \( R_v \) the absolute risk aversion ratios of, respectively, the insured and the insurer. For damages higher than \( \hat{X} \), marginal indemnities are given by

\[
\hat{C}''(\alpha_i X) = \frac{g_X R_u}{R_u + (1 + \lambda) R_v}.
\]

(ii) The optimal contract presents a disappearing deductible for losses lower than \( \hat{X} \) if the insurer is risk-neutral and an upper limit for losses beyond a level \( X \), with \( \hat{X} < X \).

If the insurer is risk averse, the coverage may display a coinsurance rate less than one for damages beyond the deductible and an upper limit of coverage.

Equation (6) is close to the one that Raviv (1979) obtained in a model with one insurable risk and to that obtained by Gollier (1996) with background risk. Actually, in our model, the risk of bad reputation is uninsurable and it depends positively on the insurable risk (we have \( g_X > 0 \)). Thus we should expect that the insured firm accepts to pay for a higher coverage of the first risk in order to protect herself against her background risk if she is prudent in the sense of Kimball (1990). Here, we obtain such a kind of result, but prudence is not necessary. Indeed in our model the second risk, \( g \), is completely defined by the first one, \( X \), so that for a given \( x_i \), both variables have the same distribution. Formally, the fact that the insured firm asks for more insurance than in a case without reputational effect is illustrated by the presence of \( g_X \), positive, at the numerator of \( C(\cdot) \). It is as if the insured firm would bear an individual “aggregate” risk, \( \alpha_i X + g(X, x_i) \), which cannot be completely insured. Besides, due to the presence of the uninsurable reputational risk, indemnities can increase with \( X \) even if the insurable loss
borne by the firm (her contribution to the Fund) is fixed and equal to $\alpha_t \hat{X}$. This result is also due to the positive correlation$^{15}$ between $g$ and $\alpha_t X$.

What is radically different from the literature on background risk is that we are dealing with catastrophe risks. Hence an insurer whose portfolio contains the aggregate risk of the Fund bears an additional risk of insolvency following a catastrophe that he has accepted to cover. Besides, empirical facts show that reinsurance groups that accept to cover pollution damages are asking for high insurance premia, which entail high risk premia. It is often argued that the management of large risks entails additional transaction costs, due to risks of insolvency or to the complexity of audits and of claims settlements. This may justify the significant increase in the price of classical insurance. In such an economic environment, it is unreasonable to assume that the insurer behaves as a risk-neutral agent. He is more likely to be risk averse and his administrative costs related to the management of catastrophe risks are sufficiently high to argue that, in most cases, the optimal insurance contract displays coinsurance between the insurer and the insured firm beyond a deductible level. In other terms, a disappearing deductible, which induces that indemnities increase more rapidly than the loss, is seldom the best contract. This result is rather intuitive since such a contract would compell the insurer to pay really high indemnities in the case of a catastrophic event. Figure 1 displays the coinsurance contract.

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Figure 1 about here

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It is still interesting to notice that a contract with coinsurance beyond a deductible may also be the best risk sharing when the insurer bears convex administrative costs, as shown by Raviv (1979). If the convexity assumption is not the most plausible when dealing with classical risks such as car- or house-insurance risks, it is much more closer to

$^{15}$For $g_X = 0$, we would have $\hat{C}''(.) = 0$ for any $X$ larger than $\hat{X}$ and $\hat{X} = \bar{X}$. 

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reality when we are focusing on large risks. Consequently, convex costs may also explain the optimality of coinsurance in the management of large risks\textsuperscript{16}.

Another important result of this section is given in Proposition 3.

**Proposition 3**

(i) The optimal level of prevention $e^C_i$ satisfies the following first order condition:

\[
1 = - \int_0^L (g_{e_i} + Q_{e_i}) \cdot u'(w^C_f(X, \hat{X})) f(X/e^C) dX \\
+ \alpha_i \int_0^L (1 + \frac{gX}{\alpha_i} - C'(\alpha X)) \cdot u'(w^C_f) F_{e_i}(X/e^C) dX \\
+ \alpha_i \int_0^L \frac{gX}{\alpha_i} - \tilde{C}'(\alpha_i X) \cdot u'(\tilde{w}^C_f(X/e^C) dX \quad (8)
\]

(ii) When the insurer can obtain information on the risk-reducing activities of the firm, the optimal level of prevention decided by the firm increases compared to a situation where no insurance is available.

This last result is not surprising. Here, the insurer can obtain information about the level of prevention decided by the firm: When a given boat is chartered, its capacity and its safeness become common knowledge because maritime authorities diffuse the results of the control. Consequently, the insurer is able to define a premium which depends on the level of prevention choosen by the oil firm\textsuperscript{17}. If insurance is available, an increase in the level of prevention decreases the level of the premium. In our model, an increase in prevention has also an effect on the marginal indemnities through its impact on the non insurable risk. Indeed, the marginal level of the bad reputation risk $g$ appears in

\textsuperscript{16}This assumption is not retained here. With a cost function more general than the one we are using, the parameter $\lambda$ would be replaced by the first derivative of the cost function and the second derivative would appear at the denominator of Equation (6).

\textsuperscript{17}In other words, the insurance premium depends on the safeness of the chartered boat in our approach.
C'(αiX). Hence as in standard models with complete information on prevention, the firm improves the prevention when she has access to insurance.

Finally, when only standard insurance is available, insurers may ask for high risk premia for accepting to manage a catastrophe risk and the optimal contract displays some coinsurance: as the damage increases, oil firms are less covered and have to bear more and more residual risk. In the next section, the issue is to find complementary mechanisms that are able to diversify risks over a wider range of individuals and to transfer risk to agents such as financial investors. In such a manner, it will be possible to reduce the residual risk borne by the firm after (standard) insurance and to increase available funds for victims in case of an accident.

3.3 Providing a better hedging strategy through capital markets

A more complete hedging strategy would consist in combining several coverage instruments. Doherty and Dionne (1993) and Mahul (2002) provide such an approach by dividing the risk into two components: an idiosyncratic risk, which can be related to the specific activities of a given firm, and a systematic risk, related to the risk of the industry as a whole. While the individual risk can be insured by a standard insurance policy, the systematic risk is managed through a participating contract. A participating contract is a policy with a variable premium based on the realized systematic loss. In a second stage, the variability of the insurance premium is hedged either through standard insurance or thanks to adequate financial instruments.

Our problematic is different from the ones of Doherty and Dionne (1993) and Mahul (2002) because 1) the oil industry does not bear an insurable idiosyncratic risk since the effect of bad reputation, which plays this role, is non insurable, 2) The individual risk of the oil firm is correlated to the risk of the Fund, while in the previous analyses both are independent, and 3) prevention is absent from these models while it plays an important role in our work.
Now, assume that the oil Fund which the firms belong to has to pay for all oil spills, whatever their size. This means that no upper limit of compensation exists ($\bar{X}$ does no longer hold) and, as a direct consequence, that oil firms are no longer protected by limited liability. Nevertheless, the firm can still transfer part of her risk to an insurer, and she can also invest on financial markets in order to cover losses (contributions to the Fund) in excess of the insurance coverage\textsuperscript{18}. Here we want to limit the implication of the insurer in the coverage of large risks in order to mitigate his bankruptcy risk. We take as given the fact that risk aversion may lead the insurer to offer a contract with an upper limit of insurance when dealing with catastrophe risks. We denote $I(.)$ the indemnity schedule and $\bar{X}$ the level of damage such that any contribution higher than $\bar{X}$ induces the same indemnity. Formally, when an oil spill occurs and after having contributed to the Fund, the oil firm obtains an indemnity $I(\alpha_i \bar{X})$ if her contribution is less than $\alpha_i \bar{X}$ and the fixed amount $\bar{I} = I(\alpha_i \bar{X})$ for any larger contribution.

Still assume that the firm can sell to an external investor a part $\beta$ of her residual risk minus the deductible\textsuperscript{19}, which is always to be borne by the firm in order to avoid some moral hazard problems\textsuperscript{20}, for any damage higher than $\bar{X}$: $\alpha_i X - \bar{I} - D^{\beta}$. The price of this risk transfer is denoted $\pi$: It depends on $\beta$ and also on the level of prevention $e_i$ adopted by the firm. In this model, financial markets can obtain some information about environmental policies adopted by the firms\textsuperscript{21}. We have $\pi = \pi(\beta, e_i)$ and it satisfies the properties $\pi_{\beta} > 0$ and $\pi_{e_i} < 0$.\textsuperscript{22} Lastly, the insurer’s unit cost of insurance is now $\delta$ and

\textsuperscript{18}Another way to deal with such a configuration would be to let firms be protected by limited liability, that means that individual contributions are still limited to $\alpha_i \bar{X}$. Nevertheless the Fund would have to pay for all damages so that it would have strong incentives to go on financial markets in order to find the additional needed funds. However such a configuration would have no impact on the oil firms in terms of prevention, so that we do not consider it here.

\textsuperscript{19}By buying and selling puts and calls of appropriate underlying securities. See the discussion at the end of this section.

\textsuperscript{20}This allows us to avoid some discontinuity of the indemnity schedule at point $D^{\beta}$ (see Figure 2).

\textsuperscript{21}See Lanoie et al. (1998) for details about how those informations circulate on financial markets.

\textsuperscript{22}The decrease of $\pi$ as prevention increases reflects the fact that financial investors are sensitive to the environmental policies adopted by the firms and they take them into account when they evaluate
it depends on $\beta$: if the insured commits to cover the worst states of nature on financial markets, the insurer takes into account this information when evaluating the insurance premium. Such a behavior implies that the firms communicate with the insurer on her financial strategy. From an empirical point of view, this is rather usual when looking at the pollution insurance market. Insurers ask for more and more informations about the risk-reducing activities of the firms and firms collaborate most always in order to obtain some adequate coverage. In our model, the consequences of a catastrophe are now split between the insurer and the financial markets. As a direct consequence, the costs of risk management for the insurer are lower than in the previous case because of a decrease in the risk premium. Formally, we have: $\delta(\beta) > 0$, $\delta(0) = \lambda$ and $\delta(\beta) < \lambda \forall \beta > 0$.

The maximization program of the oil firm becomes

$$
\max_{(I, \beta)} R^\beta = \int_0^X u(w_i - \alpha_i X - Q^\beta + I(\alpha_i X) - \pi(\beta, e_i)) f(X/e) dX \quad (9)
$$

$$
+ \int_X^L u(w_i - \alpha_i X - Q^\beta + \bar{I} + \beta [\alpha_i X - D^\beta - \bar{I}]
-g(X, \bar{x}_i) - \pi(\beta, e_i)) f(X/e) dX - e_i
$$

subject to

$$
\int_0^X v(W + Q^\beta - (1 + \delta(\beta)) I(\alpha_i X)) f(X/e) dX
+ v(W + Q^\beta - (1 + \delta(\beta)).\bar{I})(1 - F(X/e)) \geq v(W),
$$

with $Q^\beta = (1 + \delta(\beta))E[I(\alpha_i X)]$ the insurance premium. The firm has to choose the combined hedging contract $(I(\cdot), \beta)$ that maximizes her expected net revenue subject to the participation of the insurer.

---

the riskyness of their activities (see Cormier and Magnan (1997)).
Proposition 4

(i) The optimal indemnity function displays a positive deductible $D^\beta$. Marginal indemnities for losses between the deductible level and the bound $\alpha_i X$ are given by:

$$I''(\alpha_i X) = \frac{\left( 1 + \frac{\alpha X}{\alpha_i} \right) R_u}{R_u + (1 + \delta(\beta)) R_v},$$

with $R_u$ and $R_v$ the absolute risk aversion ratios of, respectively, the insured and the insurer.

(ii) For given risk attitudes of both agents and positive hedging from the financial market, the optimal coverage is higher than the one obtained when only standard insurance is available: $D^\beta < D$ and $I''(\alpha_i X) > C''(\alpha_i X)$ for any loss $\alpha_i X$ partially covered and less than $\alpha_i X$.

(iii) An increase in the financing of large losses by capital markets increases standard insurance of small and medium losses.

Point iii) enhances the fact that firms should use the wide diversification capability of financial markets to manage the potential large consequences driven by catastrophe risks and they should buy standard insurance for small and medium losses. In the specific framework of the oil industry, what we commonly call the 1992 IOPC Fund is composed of two distinct funds. The first one, the general Fund, is dedicated to the payment of the current administrative costs and to the compensation of small oil spills (less than 4 millions SDRs with one Special Drawing Right = US$ 1.46375 on 5 May 2004), while the main claims Fund is dedicated to large oil spills. Finally the general Fund should negotiate some coverage conditions offered by standard insurers, while the main claims Fund should rather be managed through interventions on capital markets.

Figure 2 displays an example of an optimal combined contract.
For convenience we use in what follows the notation \( g(X, \bar{e}_i) \equiv g \) and \( \pi(\beta, e_i) \equiv \pi \).

**Lemma 1** Partial financial hedging is optimal if and only if

\[
\pi_\beta \int_0^L u'(w^\beta_f) f(X/e) dX = \int_0^\infty I^*_\beta(\alpha_i X) u'(w^1_f) f(X/e) dX + \int_0^L (\alpha_i X - D^3 - \bar{T}) u'(w^2_f) f(X/e) dX
\]

\[
- \int_0^L Q^\beta \beta u'(w^\beta_f) f(X/e) dX,
\]

Equation (11) is obtained by differentiating (9) with respect to \( \bar{e}_i \). (Partial) external financing is optimal if the expected marginal cost of an increase in \( \bar{e}_i \) (left-hand-side-term) equals the expected marginal benefit, obtained thanks to an increase in the coverage of the small and medium losses (first member in the right-hand-side-term), to the direct increase of the coverage of large losses (second member) and to the decrease of the price of standard insurance (third member).

Lastly, we have to discuss the level of prevention adopted by the firm in the case of a joint hedging contract. A differentiation of (9) with respect to \( e^\beta \), the level of prevention in this model, and integrations by part lead to the following first order condition:

\[
1 = - \int_0^L \left( g_{e_i} + \pi_{e_i} + Q_{e_i}^\beta \right) u'(w^\beta_f) f(X/e) dX
\]

\[
+ \alpha_i \int_0^L \left( 1 + \frac{g_{X/e_i}}{\alpha_i} - I^*\pi(\alpha_i X) 1_{\{X \leq \bar{X}\}} - \beta \right) 1_{\{X > \bar{X}\}} u'(w^\beta_f) F_{e_i}(X/e) dX
\]

**Proposition 5** The level of prevention adopted by the firm is higher than the one obtained in the model with standard insurance when the opportunity of announcing her risk-reduction policy to markets induces easier access to external financing \( \pi_{e_i} < 0 \).
4 Economic implications

[to be completed] The new firms’ management of risks tries to encompass all types of risks. Firms have to cope with numerous sources of uncertainties, linked to the production process, to unanticipated market evolutions, non expected internal organization issues and also with uncertainties related to the existence of large risks. Large risks are often catastrophe risks, with random events the frequency of which may be relatively low, but that induce very large economic consequences, irreversible ecological damages and sometimes loss of human lives. This is the case for the maritime transport of oil. To manage oil spills, the 1992 IOPC Fund calls for ex post contributions by each oil firm the state of which is member of the Fund. However, no insurance mechanism is designed and only the mutuality principle is applied: The individual contribution corresponds to a percentage of the aggregate risk of the Fund. Because of the limited number of members and also of the huge financial consequences induced by some oil spills, the aggregate risk cannot be fully spread across the oil firms. Hence it is useful to think about other diversification and/or coverage instruments that would help to smooth the payments of firms through time and also to increase the funds available for compensation. In this paper, we have shown that transferring part of the aggregate risk, namely the part related to catastrophic losses, to investors that have access to capital markets makes standard insurance of small and medium oil spills less costly. The mixed strategy, which consists in using the properties of standard insurance for risks that are reasonably insurable and the wide capability of financial markets to diversify risk across many people in the world for catastrophic losses, seems to be a good compromise. Moreover if firms can send to the markets signals on their environmental policies, financing hedging creates additional incentives to invest in risk-reducing activities....
Figure 1. Optimal Compensation Function when only standard insurance is available
Figure 2. An example of optimal hedging strategy
(At fixed premium)
APPENDIX

Proof of Proposition 1

Recall that \( w_f = w_i - \alpha_i X - g(X, \bar{x}_i), \)
\( \bar{w}_f = w_i - \alpha_i \bar{X} - g(X, \bar{x}_i), \)
\( g_{e_i} = g_{e_i}(X, \bar{x}_i), \)
\( g_X = g_X(X, \bar{x}_i). \)
A differentiation of (1) with respect to \( e_i \) leads to:

\[
R_{e_i} = - \int_0^X g_{e_i}(X, \bar{x}_i).u'(w_f)\,f(X/e)\,dX - \int_0^L g_{e_i}(X, \bar{x}_i).u'(\bar{w}_f)\,f(X/e)\,dX
+ \int_0^{\bar{X}} u(w_f)\,f_{e_i}(X/e)\,dX + \int_0^{\bar{X}} u(\bar{w}_f)\,f_{e_i}(X/e)\,dX - 1
\]

With \( F_{e_i}(0, e) = F_{e_i}(L/e) = 0, \) integrations by part of the third and fourth term induce that:

\[
R_{e_i} = - \int_0^{\bar{X}} g_{e_i}.u'(w_f)\,f(X/e)\,dX - \int_0^L g_{e_i}.u'(\bar{w}_f)\,f(X/e)\,dX
+ \int_0^{\bar{X}} (\alpha_i + g_X).u'(w_f)F_{e_i}(X/e)\,dX + \int_0^L g_X.u'(\bar{w}_f)F_{e_i}(X/e)\,dX - 1
\]

If an interior solution exists, then it satisfies \( R_{e_i} = 0. \) Point i) is demonstrated.

For Point ii), the effect on prevention of a variation in the maximum amount of loss covered by the Fund is given by

\[
\frac{d e_i}{d\bar{X}} = \frac{\alpha_i}{R_{e_i,e_i}} \left[ \int_0^\bar{X} g_{e_i}.u''(\bar{w}_f)\,f(X/e)\,dX + u'(w_i - \alpha_i \bar{X} - g(\bar{X}, \bar{x}_i)).F_{e_i}(\bar{X}/e) - \int_0^L \int_0^\bar{X} g_X.u''(\bar{w}_f)F_{e_i}(X/e)\,dX \right],
\]

(13)

with \( R_{e_i,e_i} \) the derivative of \( R_{e_i} \) given by (3) with respect to \( e_i. \)

Notice that the higher the prevention, the less the bad reputation risk: \( g_{e_i} < 0. \)
Besides, an increase in the aggregate loss \( X \) of the Fund deteriorates the reputation of
all firms, so that \( g_X > 0 \). By assumption we also have that \( F_{e_i} \) is positive. Finally, the numerator of (13) is strictly positive for a risk-averse, or risk-neutral, oil firm. The denominator is obtained thanks to a differentiation of (3) w.r.t. \( e_i \). With \( g = g(X, \bar{x}_i) \) and \( w_f(X, \bar{X}) = w_i - g(X, \bar{x}_i) - \alpha_i X.1_{\{X \leq \bar{X}\}} - \alpha_i \bar{X}.1_{\{X > \bar{X}\}} \) we have:

\[
R_{e_i e_i} = - \int_0^L g_{e_i e_i}.u'(w_f(X, \bar{X}))f(X/e)dx + \int_0^L g_{e_i}^2.u''(w_f(X, \bar{X}))f(X/e)dx
\]

\[
+ \int_0^L g_{x e_i}.u'(w_f(X, \bar{X}))F_{e_i}(X/e)dx
\]

\[
- \int_0^\bar{X} g_{e_i}.u'(w_f(X, \bar{X}))f_{e_i}(X/e)dx
\]

\[
- \int_0^\bar{X} (\alpha_i + g_X).g_{e_i}.u''(w_f)F_{e_i}(X/e)dx - \int_\bar{X}^L g_x.g_{e_i}.u''(w_f)F_{e_i}(X/e)dx
\]

\[
+ \int_0^\bar{X} (\alpha_i + g_X).u'(w_f)F_{e_i e_i}(X/e)dx + \int_\bar{X}^L g_x.u'(w_f)F_{e_i e_i}(X/e)dx
\]

From the definition (2) of \( g(X, \bar{x}_i) \), we have that \( g_{e_i} = g_{x e_i} = 0 \). Finally, an integration by part of the third line leads to:

\[
R_{e_i e_i} = - \int_0^\bar{X} g_{e_i e_i}.u'(w_f(X, \bar{X}))f(X/e)dx + \int_0^\bar{X} g_{e_i}^2.u''(w_f(X, \bar{X}))f(X/e)dx
\]

\[
- 2 \int_0^\bar{X} (\alpha_i + g_X).g_{e_i}.u''(w_f)F_{e_i}(X/e)dx - 2 \int_\bar{X}^L g_x.g_{e_i}.u''(w_f)F_{e_i}(X/e)dx
\]

\[
+ \int_0^\bar{X} (\alpha_i + g_X).u'(w_f)F_{e_i e_i}(X/e)dx + \int_\bar{X}^L g_x.u'(w_f)F_{e_i e_i}(X/e)dx
\]

If the second order conditions are satisfied, then \( R_{e_i e_i} \) is negative. By assumption, we have \( F_{e_i e_i} \leq 0, g_X > 0 \) and \( u'' < 0 \). Besides, \( g_{e_i e_i} \) is equal to \( p_{e_i e_i}.(g(X, x_i) - g(X, 0)) \) (see 23Function \( 1\{\cdot\} \) is the indicator function, taking value one if the condition into brackets is satisfied, zero otherwise.

23
Equation (2)). Function \( g(X,) \) is increasing in \( x_i \) and \( p_{e_{i},e_{i}} \) is positive or equal to zero, so that \( g_{e_{i},e_{i}} \) is positive. Finally \( R_{e_{i},e_{i}} \) is negative and \( dc_i/d\bar{X} \) given by (13) is positive. Point ii) of Proposition 1 is demonstrated.

**Proof of Proposition 2**

The optimality conditions related to optimal control that must be satisfied are

\[
\begin{align*}
(i) \quad H_z &= -\mu'(X) \\
(ii) \quad H_\mu &= z(X) \\
(iii) \quad z(0) &= 0 \\
(iv) \quad z(L) &= v(W) 
\end{align*}
\]

and \( H_C = 0, \forall X \) such that \( 0 < C(\alpha_i X) < X \). From (5) we have \( H_z = 0 \) so that \( \mu \) is constant. Conditions (ii), (iii) and (iv) are also satisfied. Besides \( f(X/e) \) is, by definition, always positive. Hence it is possible to work with the simplified Hamiltonian \( H^* = H/f(X/e) \). We have for any \( X \) such that \( 0 < C(\alpha_i X) < X \):

\[
H_C^* = 0
\]

\[
\iff u'(w_f^C(X,\bar{X})) - \mu(1 + \lambda)v'(W_f^C) = 0
\]

with \( w_f^C(X,\bar{X}) = w_i - \alpha_i X.1_{\{X \leq \bar{X}\}} - \alpha_i \bar{X}.1_{\{X > \bar{X}\}} + C(\alpha_i X) - Q - g(X,\bar{x}_i) \) and \( W_f^C = W + Q - (1 + \lambda)C(\alpha_i X) \).

First, we have to show that the optimal contract displays a positive deductible. Let us define as \( J(X) \) the function given by (14) and evaluated at \( C(\alpha_i X) = 0 \) and \( K(X) \) the same function but evaluated at \( C(\alpha_i X) = \alpha_i X \). By differentiationg them w.r.t. \( X \) it is easy to show that \( J(X) \) is increasing in \( X \) and \( K(X) \) is decreasing. Moreover, both functions are equal at point \( X = 0 \). Denote them \( L : L = u'(w_i - Q) - \mu(1 + \lambda)v'(W + Q) \). Two cases must be considered : either \( L \) is negative or \( L \) is positive (the trivial case for which \( L = 0 \) is not analyzed).

\( \blacklozenge \) \( L > 0 \)

Since \( J \) is increasing in \( X \), \( L \) is the smallest value it can take. Thus \( J \) is always
positive and $C(\alpha_iX) = 0$ is never optimal\textsuperscript{24}. Besides, $K$ is decreasing in $X$. Then it exists a positive level of damage $\hat{X}$ such that $K$ is positive on $[0, \hat{X}]$ and $C(\alpha_iX) = \alpha_iX$ is optimal on this interval. For damages higher than $\hat{X}$, $K$ becomes negative: from this point, coverage must be constant and an upper limit of insurance is optimal.

\[ \diamond L < 0 \]

In this case, $K$ is always negative and full coverage is never optimal. Besides, it exists a level of damage $D$ such that $J$ is negative on $[0, D]$ and presents partial coverage for any damage higher than $D$. A positive deductible is optimal.

Following Raviv (1979), we can show that, at fixed insurance premium, an upper limit is always stochastically dominated by pure coinsurance when insurance is costly. The intuition is that the risk averse insured prefers a transfer of indemnities of small damages to higher ones when insurance is costly. In the same spirit, a deductible contract dominates a pure coinsurance contract in the sense of second order stochastic dominance (Gollier and Schlesinger (1996)). Hence, the optimal contract displays a strictly positive deductible as long as the marginal cost of insurance $\lambda$ is positive.

Second, we have to define the optimal marginal indemnities beyond the deductible level. This is done first on $]D, \hat{X}[$ and second on $]\hat{X}, L[$. By differentiating Equality (14) w.r.t. $X$ on $]D, \hat{X}[$ and using it to define $\mu$ we must have, for any loss partially covered on $]D, \hat{X}[$:

\[ (-\alpha_i + \alpha_i C''(\alpha_iX) - g_X).u''(w_f^C) + (1 + \lambda)^2 \cdot \alpha_i . C''(\alpha_iX) . \mu . v''(W_f^C) = 0 \]

\[ \iff C''(\alpha_iX) = \frac{(1 + \frac{g_X}{\alpha_i}) u''(w_f^C)}{u''(w_f) + (1 + \lambda) \cdot \frac{v''(W_f^C) . v'(w_f^C)}{v'(W_f^C)}} \]

\[ \iff C''(\alpha_iX) = \frac{(1 + \frac{g_X}{\alpha_i}) R_u}{R_u + (1 + \lambda) . R_v} \]

Equation (6) in Point i) is demonstrated. If the insurer is risk neutral we have $R_v$ equal to zero and $C'' = 1 + \frac{g_X}{\alpha_i}$. Since all terms are positive, the slope of the compensation $\nabla H_{CC} = u''(w_f^C) + \mu (1 + \lambda)^2 v''(W_f^C) < 0$. The second order conditions are satisfied and the result holds.

\[ \text{25} \]
function for any damage partially covered is larger than one. The deductible disappears progressively as the damage increases.

Equation (7) in Point i) is obtained thanks to an identical reasoning, but with $X$ in $] \hat{X}, L \lbrack$ and $\hat{u}_f^C = w_i - \alpha_i \hat{X} + C(\alpha_i X) - Q - g(X, \bar{X})$. By assumption, we have that $g_{XX}$ is negative, so that marginal indemnities decrease with $X$ for losses higher than $\hat{X}$. Consequently, from a level of damage $X$ larger than $\hat{X}$, marginal indemnities are close to zero and the compensation function displays an upper limit.

If the insurer is risk averse and asks for a large risk premium, which means that $\lambda$ is large, the value of $C'(\alpha_i X)$ may be less than one so that coinsurance for any partially indemnified loss on $] D, \hat{X} \lbrack$ is optimal. Point (ii) of Proposition 2 is demonstrated.

Proof of Proposition 3

Point i) is obtained thanks to a differentiation of (4) w.r.t. $e_i$. We denote $e_i^C$ the solution of the first order condition

$$ R_{e_i}^C = 0 $$

$$ \Leftrightarrow \quad 1 = - \int_0^L (g_{ei} + Q_{ei}).u'(w_f^C(X, \hat{X}))f(X/e^C)dX + \int_0^{\hat{X}} u(w_f^C)f_{ei}(X/e^C)dX $$

$$ + \int_{\hat{X}}^L u(w_f^C)f_{ei}(X/e^C)dX \quad (15) $$

Integrations by part of the two last terms give:

$$ \int_0^{\hat{X}} u(w_f^C)f_{ei}(X/e^C)dX + \int_{\hat{X}}^L u(w_f^C)f_{ei}(X/e^C)dX $$

$$ = \quad \alpha_i \int_0^{\hat{X}} (1 + \frac{gX}{\alpha_i} - C'(\alpha_i X))u'(w_f^C)F_{ei}(X/e^C)dX $$

$$ + \alpha_i \int_{\hat{X}}^L \frac{gX}{\alpha_i} - \hat{C}'(\alpha_i X))u'(w_f^C)F_{ei}(X/e^C)dX $$
By replacing the right-hand-side term of this equation in (15), we obtain, at optimum:

\[
1 = - \int_0^L (g_{e_i} + Q_{e_i}) \cdot u'(w_f(X, \hat{X})) f(X/e^C) dX \\
+ \alpha_i \int_0^\hat{X} \left((1 + \frac{gX}{\alpha_i}) - C'(\alpha_i X)\right) u'(w_f(X/e^C)) dX \\
+ \alpha_i \int_{\hat{X}}^L \left((\frac{gX}{\alpha_i} - \hat{C}'(\alpha_i X)) u'(w_f(X/e^C)) \right) dX
\]  

(16)

Point i) is demonstrated. Point ii) is obtained thanks to a differentiation of (16) w.r.t. \(e_i\) and \(C\):

\[
\frac{de_i}{dC} = \frac{1}{-R_{e_i}^C} \left[ - \int_0^L (g_{e_i} + Q_{e_i} - (1 - Q_C) \cdot u''(w_f(X, \hat{X})) f(X/e^C) dX \\
- \int_0^L Q_{e,C} \cdot u'(w_f(X, \hat{X})) f(X/e^C) dX \\
+ \alpha_i \int_0^{\hat{X}} \left((1 + \frac{gX}{\alpha_i}) - C'(\alpha_i X)\right)(1 - Q_C) \cdot u''(w_f(X/e^C)) dX \\
+ \alpha_i \int_{\hat{X}}^L \left((\frac{gX}{\alpha_i} - \hat{C}'(\alpha_i X)) (1 - Q_C) \cdot u''(w_f(X/e^C)) \right) dX \right]
\]  

(17)

(18)

(19)

Marginal compensations \(C'(\alpha_i X)\) are always lower than or equal to \(1 + \frac{ax}{\alpha_i}\) at optimum (see Equation (6)), while \(\hat{C}'(\alpha_i X)\) is always lower than or equal to \(\hat{C}'(\alpha_i X)\) (see Equation (7)). The premium \(Q\) is equal to \(\int_0^L (1 + \lambda)C(X, \hat{X}) f(X/e^C) dX\), with \(C(X, \hat{X}) = C\) on \([0, \hat{X}]\) and \(C(X, \hat{X}) = \hat{C}\) on \(\hat{X}, L]\); Consequently, \(Q_{e_i} = \int_0^L (1 + \lambda) C_{e_i}(X, \hat{X}) f_{e_i}(X/e^C) dX = -\alpha_i \int_0^L (1 + \lambda) C_{X}(X, \hat{X}) F_{e_i}(X/e^C) dX\), which is positive, \(Q_C = \ldots\)
and $Q_{e,C}$ equals zero. Equation (19) becomes:

$$
\frac{d e}{d C} = \frac{\lambda}{-R_{e,C}} \cdot \left[ \int_0^L (g_{e_i} + Q_{e_i}).u''(w_f^C(X, \bar{X}))f(X/e^C)dX 
- \alpha_i \int_0^L (1 + \frac{gX}{\alpha_i} - C'(\alpha_i X)).u''(w_f^C)F_{e_i}(X/e^C)dX 
- \alpha_i \int_0^L \frac{gX}{\alpha_i} - \tilde{C}'(\alpha_i X)).u''(w_f^C)F_{e_i}(X/e^C)dX \right]
$$

The second order conditions of this problem are satisfied (the computation is similar to the one presented in the proof of Proposition ??), so that $R_{e,C}$ is negative. Finally, $\frac{d e}{d C}$ is positive and Point ii) of Proposition 1 is demonstrated.

Proof of Proposition 4

The control variable is $I(\alpha_i X)$ and the state variable is $z(X) = \int_0^X v(W + Q^\beta - (1 + \delta(\beta))I(\alpha_i X))f(t/e)dt$. The simplified Hamiltonian of Program (9) is

$$
H^{\beta*} = u(w_f^1).1_{\{X \leq \bar{X}\}} + u(w_f^2).1_{\{X > \bar{X}\}} - e_i + \gamma(X)v(W_f^{\beta*}),
$$

with $\gamma(X)$ the Lagrange function, $w_f^1 = w_i - \alpha_i X - Q^\beta + I(\alpha_i X) - g(X, \bar{x}_i) - \pi(\beta, e_i)$, $w_f^2 = w_i - \alpha_i X - Q^\beta + \bar{T} + \beta [\alpha_i X - \bar{T}] - g(X, \bar{x}_i) - \pi(\beta, e_i)$ and $W_f^{\beta*} = W + Q^\beta - (1 + \delta(\beta))I(\alpha_i X)$. Function $1_{\{X > \bar{X}\}}$ is the indicator function, which takes value 1 when the condition into brackets is satisfied, zero otherwise. Still here, the Lagrange function is a constant. We have for any $X$ in $]0, \bar{X}[\ such that $0 < I(\alpha_i X) < X$:

$$
H_f^{\beta*} = 0
\iff u'(w_f^1) - \gamma(1 + \delta(\beta))v'(W_f^{\beta*}) = 0
$$

(21)

First, we have to show that the optimal contract displays a positive deductible denoted $D^\beta$. The proof is similar to that proposed for Proposition 2.

Second, by differentiating Equality (21) w.r.t. $X$ and using it to define $\gamma$ we must have, for any $X$ in $]D^\beta, \bar{X}[\ such that $0 < I(\alpha_i X) < X$,

$$
(-\alpha_i + \alpha_i. I''(\alpha_i X) - gX).u''(w_f^1) + (1 + \delta(\beta))^2.\alpha_i. I''(\alpha_i X).\gamma.v''(W_f^{\beta*}) = 0,
$$

(28)
\( I'(\alpha_iX) = \frac{(1 + \frac{\partial \delta}{\partial \alpha_i}).u''(w^1_{\lambda})}{u''(w^1_{\lambda}) + (1 + \delta(\beta)).\frac{\partial u'(W^0_{\lambda})}{\partial W^0_{\lambda}}} \)

\( I'(\alpha_iX) = \frac{(1 + \frac{\partial \delta}{\partial \alpha_i}).R_u}{R_u + (1 + \delta(\beta)).R_v} \) \hspace{1cm} (22)

Point i) is demonstrated. For point ii), we know that \( \delta \) is decreasing in \( \beta \) and that \( \delta(0) = \lambda \). The marginal indemnities \( I^* \) and \( C^* \) (given by (6)) differ from the term \( \delta \) present at the denominator of \( I^* \). Hence \( I^* \) is always higher than \( C^* \) when \( \beta \) is positive.

Point iii) is immediate. From (22), marginal indemnities increase as \( \beta \) increases.

**Proof of Proposition 5**

The structures of Condition (8) and (12) differ only by the term \(-\pi_e \int_0^L u'(w^\beta_{\lambda})f(X/e)dx\), which is positive. By taking this positive term into account when looking at the expected marginal benefit of prevention and by applying the same reasoning as in the proof of Point ii) of Proposition 3, Proposition 5 is demonstrated.

**References**


